Essays on Auctions

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Contents

CHAPTER 1

Introduction

This thesis is a collection of papers on auctions. Auctions are mechanisms allocating goods (e.g. electric power, CO_2 abatements, timber, and various asset auctions) to bidders (e.g. individuals, firms, and institutions). Auction theory is based on the game theoretical models, which are successful in the laboratory (see Kagel, 1995) and in the field (see for example Hendricks, Porter and Wilson, 1994). There are practical considerations (e.g. the complexity of the auction, time constraints of the bidders) which have to be taken into account when designing the auction. To satisfy these needs, complicated auctions are often employed. In these auctions, there are tiny details which an economist needs to carefully study to predict the final outcome. This thesis focuses on four dimensions of auction design which are important for economic transactions: the ending rule, scheduling of the auctions, transmission of the bids and information feedback.

The structure of the thesis is as follows. The remainder of this introduction introduces the four dimensions of auction design studied in the thesis and outlines the chapters of the thesis, the research questions they pose and their main conclusions. Part I of this thesis, which deals with scheduling, the ending rule and transmission of bids in Internet auctions, consists of chapters 2, 3 and 4. Part II of this thesis focuses on information feedback in sealed-bid auctions (chapters 5 and 6).

1.1. Ending rule

When designing an auction, the auctioneer may choose between sealed-bid and open mechanism. While a sealed-bid auction takes only one round, an open auction lasts for longer. An open auction gives bidders time to discover their optimal strategies. On the other hand side, there is a concern whether it would ever end. To solve the problem, the auctioneer often introduces an activity rule that prevents a bidder from holding back by tying a bidder's eligibility in future rounds to its activity in the current round (see Milgrom, 2004). Clearly, such a solution is not desirable when it is hardly possible to gather all the potential bidders in one place or time (e.g. in Internet auctions). In those situations, the activity rule is not imposed and the problem of the possible infinite duration of the auction is solved by the introduction of an ending rule. This thesis studies some of the possible ending rules and their impact on the bidding behavior.

1.2. Scheduling of the auctions

There are various ways of auctioning several goods. The goods may be auctioned all at once or in separate sequential auctions. The choice of the schedule depends on the auctioneer's goal. Auctioning all the goods at once is attractive for an impatient auctioneer. Sequential auctions may attract more bidders. Furthermore, they appear to be easier to organize. There are also theoretical considerations which might be taken into account when scheduling the auctions. In particular, different scheduling of the auctions may result in different outcomes. This thesis investigates how the choice of the schedule of Internet auctions influences the allocation of the goods.

1.3. Transmission of the bids

In most auctions, a submitted bid is received by the auctioneer for sure. In some circumstances, there might be some random factor (e.g. imperfect Internet connections, which bidders use to submit their bids) causing rejection of the bids. This thesis addresses a question whether a bidder might find it optimal to bid under uncertain bid transmission, despite being given time to bid under certain bid transmission and if so, what the consequences of such behavior are.

1.4. Information feedback

Different information feedback may be provided after the auction finishes. For example, after the auction ends, all the bidders might learn the identities of the winning bidders and the prices; alternatively only the winning bidder might be informed about the price. Since the bidders learn the feedback after the auction ends, one may think that in private-value auctions, the information feedback plays no role. On the other side, bidders may engage in counterfactual thinking, which depends on the information feedback. This thesis follows the latter approach and examines the impact of information feedback on behavior of regretful bidders.

1.5. Summary of the chapters

1.5.1. Part I

Part I of this thesis consists of chapters 2, 3 and 4, all studying Internet auctions. On the Internet, single sellers may independently run their auctions. Auctions offering identical goods are often run sequentially or simultaneously. A single auction is typically run for seven days, during which bidders are free to submit their bids. It ends at a fixed point of time that has been exogenously fixed in advance (e.g. on eBay) or at an endogenously determined point of time which depends on the bidding activity (e.g. on Amazon). Under the fixed ending rule, despite lots of bidding possibilities, bidders often bid just before the fixed ending of the auction. Under the flexible ending rule, the distribution of the bids is more spread out.

Part I of this thesis explains, the phenomenon of sniping, a common practice of placing a bid in the closing seconds of the auction with a fixed ending rule, in the context of multiple auctions selling identical independent private-value goods. Alternative explanations proposed in the literature suggested that sniping might be related to uncertainty of a bid transmission in the last minute of the auction or uncertainty in the value of the good (see Bajari and Hortacsu, 2004, for an overview). It is not obvious that with the current Internet technology bidders might face problems with sending their bids just before the end of the auction. Furthermore, many Internet auctions offer goods with the well-known value to the bidders. Part I of this thesis focuses on such auctions and explains sniping under certain transmission of all the bids.

Besides focusing on sniping, part I addresses the following questions related to Internet auctions. First, it compares different ending rules (chapter 2). Second, it studies the impact of the scheduling of the auctions on the final allocation (chapter 2, 3 and 4). Third, it investigates the relation between possible imperfections of the transmission of the bids and bidding behavior (chapter 4).

Chapter 2 analyzes fixed and flexible ending rules in sequential Internet auctions. There are two open consecutive second-price auctions. When bidders bid early in the first auction, the information on the ordering of their independent private valuations is revealed. Weak bidders (those with the third highest value or lower) conclude that, in an efficient equilibrium, they have no chance to win. Therefore, they bid their valuations in the first auction, which increases the final price and makes the revelation of the information unattractive. Under the fixed ending rule, in equilibrium, all the bidders bid at the last stage of the first auction, which gives a rational for sniping. The flexible ending rule leaves no space for sniping and hence, disables an efficient outcome. Therefore, a fixed ending time might be preferable.

Chapter 3 models simultaneous Internet auctions with certain bid transmission and a fixed ending rule. There are two simultaneous dynamic auctions offering the same good. Bidders have lots of flexibility to submit their bids, but they are restricted by the fixed ending rule. If bidders bid early, the strong bidder (the one with the highest valuation) will become the current winner and the weak bidders (those with the third highest valuation or lower) will infer that they have no chance to fight against the strong bidder and, hence, will be unwilling to bid any longer. If the weak bidders do not bid any more, the strong bidder will also have no reason to submit more bids. But if the strong bidder stops bidding, the weak bidders actually have a chance to win. All in all, an equilibrium with early bidding cannot be reached. In equilibrium, bidders bid only in the last round and an efficient outcome is not guaranteed.

Chapter 4 compares simultaneous Internet auctions to overlapping Internet auctions, assuming that the bid transmission is uncertain in the last minute of each auction. Two auctions selling identical goods overlap or coincide in time. Before the last stage, each bid is accepted with certainty. At the last stage, a bid is rejected with some positive probability. The chapter rationalizes both multiple bidding as well as sniping: there is multiple bidding to allow coordination, and there is sniping when the coordination starts late. In overlapping auctions, even if the coordination starts late, bidders always have time to safely reallocate without engaging in risky sniping. Hence, overlapping auctions may be better than simultaneous ones, as, in contrast to the former, the latter might cause inefficiencies related to risky sniping.

To conclude, part I argues that to sustain efficiency, the auctioneer should opt for sequential or overlapping auctions. Furthermore, chapter 2 shows that having a fixed ending time instead of a flexible ending time is good for efficiency. Finally, all the chapters show that the phenomenon of sniping may be attributed to the multiplicity of auctions with the same offerings.

1.5.2. Part II

Experiments investigating private-value sealed-bid auctions show that bidders bid more aggressively than the auction theory predicts (see Kagel, 1995). To describe the observed behavior, alternative models have been proposed in the literature. In these models, bidders do not only care about their monetary payoffs, but also experience emotions (e.g. regret) affecting their choices. An economist is often unable to directly observe the emotions and has to take an indirect approach to make testable predictions. In particular, one may indicate the factors important for the new models and irrelevant for the standard models. Part II takes such an approach by investigating the role of the information feedback on bidding behavior of regretful bidders in sealed-bid auctions.

Chapter 5 presents a theoretical study of the impact of information feedback on the anticipated regret and rejoicing on bidding behavior in sealed-bid auctions. On learning the outcome of the auction, a player may discover that another bid would have lead to a higher payoff. This knowledge may impart regret. Alternatively, he may realize that he has chosen the best possible bid, given the actions of his opponents. This knowledge may impart rejoicing. Chapter 5 shows that a player who is prepared to trade-off financial return in order to avoid regret and to maximize rejoicing will bid more aggressively than the standard theory suggests. The behavior depends on the information feedback, as a bidder experiences regret or rejoicing only if he is aware of the foregone opportunities.

Chapter 6 reports on the results of an experiment on the relation between public revelation on the price and bidders' behavior in private-value sealed-bid auctions. The observed dynamic behavior is affected by the experience of regret, which in turn is related with the information feedback. A subject who experiences a material loss because of bidding above the valuation tends to bid less aggressively in the next round. A subject who loses when the price is below his valuation most often bids more aggressively in the next round. When losing bidders are not provided feedback on the price, the regret driven reactions are less popular. All in all, regret seems to be an important factor of bidders' decisions.

Part 1

Internet Auctions

CHAPTER 2

On the Ending Rule in Sequential Internet Auctions

2.1. Introduction

In online auctions, buyers do not need to be available in the same time or place. Instead, they can enter their bids whenever it is convenient to them. The possible bidding time has its limit. The auction has an ending rule specified in advance. Different ending rules are used in Internet auctions. On eBay, the seller specifies a fixed ending time for the auction. All bids must be placed within the auction duration. It is possible to snipe, that is to place a bid in the closing seconds of the auction, so that the other buyers have no time to react. On Amazon, the seller determines the length of the auction, but the ending time is flexible. Whenever a bid is submitted in the last 10 minutes of the specified auction duration, the auction is automatically extended for an additional 10 minutes from the time of the latest bid. It is always possible to react to the late bid. On Yahoo, the seller can opt for the flexible ending time or choose to have the fixed ending time.

The existing empirical studies show that under the fixed ending time, there is a great deal of sniping in Internet auctions (see Bajari and Hortacsu, 2004, for an overview). Sniping cannot be explained by bidders' preferences, as under the flexible ending time, bidders submit their bids earlier. Bajari and Hortacsu (2003) and Roth and Ockenfels (2006) rationalize sniping in single-unit common-value auctions. Roth and Ockenfels also present a model of a single-unit private-value Internet auction with uncertain transmission of the late bids. In their model, sniping is one of the possible equilibria and is driven by the uncertain transmission of the late bids. Nowadays, Internet connections are very fast and there are computerized agents that snipe on behalf of the bidders. Therefore, it is desirable to present an explanation of sniping in a model with certain transmission of the late bids and private values.

This paper proves rationality of sniping in a sequential private-value Internet auction. The proposed model is reminiscent of Peters and Severinov (forthcoming), who model competing auctions selling private-value goods. In their model, all the auctions are run simultaneously and the bidding lasts as long as there is someone interested in further bidding. In the present paper, auctions are run sequentially. Peters and Severinov show that incremental bidding is an equilibrium strategy. The present paper shows that under the fixed ending time, sniping is an equilibrium strategy.

The paper is structured as follows. The next section studies sequential Internet auctions with fixed ending times. Subsection 2.2.1 introduces the model of the Internet auctions with the fixed ending times. Subsection 2.2.2 argues that sniping is the unique equilibrium strategy. Section 2.3 analyzes sequential Internet auction with flexible ending times. Subsection 2.3.1 refines the model defined in subsection 2.2.1 to allow for the flexible ending time. Subsection 2.3.2 shows that there is no efficient equilibrium under the flexible ending time. Finally, section 2.4 concludes and suggests directions for the future research.

2.2. Fixed ending rule

2.2.1. The model

There are two consecutive auctions (auction 1 and auction 2) and $N > 2$ buyers. After auction 1 ends, auction 2 starts. Each buyer i ($i = 1, ..., N$) has an independent private valuation of one (and only one) item of the good (v_i), which is distributed according to standard distribution $F(v_i)$ with density $f(v_i) > 0$ and support on $[0, 1]$. $v^{m:n}$ denotes the m^{th} highest valuation out of n bidders. $F^{m:n}(.)$ is distribution of $v^{m:n}$. $f^{m:n}(.)$ is corresponding density.

Since buyers have unit demands, a buyer who wins the good in auction 1 weakly prefers not to bid in auction 2. For clarity purposes, I assume that he is not given that choice. A buyer i is allowed to bid in the auction 2, if he does not win the good in the auction 1. Every buyer who is allowed to bid in a given auction is called a *potential buyer* of this auction. There are N potential buyers in auction 1 and $N-1$ potential buyers in auction 2.

Each auction a ($a = 1, 2$) takes two rounds. In each round t ($t = 1, 2$) of auction a, the following happens:

(1) The *current price* (p_a^t) is announced.

(2) Bids are collected from the potential buyers.

(3) *Active bidders,* the identity of the buyer whose bid equals the price and the *current winner* are indicated.

In round 1 of auction a, the current price is 0. In round 2 of auction a, the *current price* equals the highest bid submitted by buyer who is not the *current winner* or 0, if there is only one *active bidder.* In each round t of auction a, after the current price is announced, each potential buyer i can submit a positive bid $b_{a,i}^t > 0$ or not to bid at all ($b_{a,i}^t = 0$). I focus only on monotonic bidding functions, which is natural for auctions (that is $\frac{\partial}{\partial v_i} b_{a,i}^t(.) > 0$ or $b_{a,i}^t(.) = 0$). A bid of buyer i is accepted if it exceeds the current price and his previous bids submitted in this auction. Each buyer whose bid is accepted in auction a becomes an *active bidder* of auction a. He remains an active bidder until the end of auction a. Buyer i who submits the highest bid in auction a as the first one becomes a *current winner.* In case of a tie, the current winner is chosen randomly. In the end of each round, the active bidders, the current winner and the buyer whose bid equals the current price are indicated. After the last round of auction a, the current winner wins the good. The final price (p_a) is chosen on the same basis as the current price.

After auction 2 finishes, the game ends. The utility of buyer i is given by:

$$u_i(.) = \begin{cases} v_i - p_a & \text{if } i \text{ is the final winner in auction } a \\ 0 & \text{if } i \text{ does not win any good} \end{cases}$$

Let $h_a^t \in H_a^t$ be a history of the learned outcomes (i.e. current winners, current prices, identity of buyers whose bids equal to current prices and identities of all other active buyers) up to round t of auction a. Let $s_{a,i}^t \in S$ denote a status (current winner, losing bidder or a bidder who did not bid) of buyer i in round t of auction a. Then, an action of buyer i in round t of auction a is given by $b_{a,i}^t(v_i, h_a^t, s_{a,i}^t) \in R_+$. The strategy of buyer i is described by the function:

$$\sigma_i = \begin{bmatrix} b_{1,i}^1(v_i, h_1^1, s_{1,i}^1) & b_{1,i}^2(v_i, h_1^2, s_{1,i}^2) \\ b_{2,i}^1(v_i, h_2^1, s_{2,i}^1) & b_{2,i}^2(v_i, h_2^2, s_{2,i}^2) \end{bmatrix} : [0,1] \times H \times S^4 \to R_+^4$$

where H is the space of all possible histories.

Let Ω_i be the space of all possible σ_i. The beliefs of buyer i on the valuations of the opponents ($v_{-i} = \{v_1, ..., v_{i-1}, v_{i+1}, ..., v_N\}$) are summarized by $\mu_i(v_{-i}|v_i, h_a^t)$. The prior belief is that each valuation is distributed according to $F(.)$. The final outcome, including final prices and final winners for all auctions, is given by $o \in O$. The outcome function is $\varpi : \Omega \times [0,1]^N \to O$, where $\Omega = \Omega_1 \times \cdots \times \Omega_N$. The game is defined by $\Gamma_1 = \{\Omega_1, ...\Omega_N, o(.)\}$. The equilibrium concept is efficient symmetric perfect Bayesian equilibrium, where efficiency means that buyers with the two highest valuations win the goods, symmetry implies that $\sigma_i = \sigma_j$ for every $i, j \in \{1, ..., N\}$ and perfect Bayesian equilibrium is defined in a standard way.

2.2.2. Results

Analyzing the behavior in auction 2 is not interesting, as each potential bidder weakly prefers to bid the valuation in some round of auction 2. Therefore, I focus on the optimal bidding behavior in auction 1. In particular, I wish to show that in auction 1, it is optimal to snipe, where sniping is meant as not bidding in round 1 of auction 1 and bidding in round 2 of auction 1. In order to do so, I need to prove that it is not in interest of a bidder to bid early, where early bidding is defined as bidding in round 1 of auction 1, and that an equilibrium with sniping exists.

To solve the game, I focus only on strategies leading to an efficient allocation. Note that a bidder with valuation equal to 0 will not bid in any auction, as bidding above the valuation is never in his interest. Hence, his expected utility is 0. By Revenue Equivalence Theorem, if the allocation is efficient and the bidder with valuation equal to 0 obtains utility of 0, the expected prices equal to the value of the third highest valuation ($E[p_1] = E[p_2] = E[v^{3:N}]$) (see Krishna, 2002).

The optimal bidding behavior in round 2 of auction 1 depends on what happens in auction 2. As I have already argued, in an efficient equilibrium, bidders behave in such a way in auction 2 that $E[p_2] = E[v^{3:N}]$. In an efficient equilibrium, when choosing his optimal bid in auction 1, a bidder thus assumes that $E[p_2] = E[v^{3:N}]$. His behavior in auction 1 depends on his belief on the relation between his valuation and $E[v^{3:N}]$. His prior beliefs are such that he expects that with a positive probability his valuation is higher than $E[v^{3:N}]$. In round 2 of auction 1, he might have already updated his beliefs in such a way that he knows that his valuation is not higher than $E[v^{3:N}]$ with certainty. In such a case, he realizes that in auction 2, where $E[p_2] = E[v^{3:N}]$, he has no chance for a positive transaction. If so, he weakly prefers to bid his valuation in round 2 of auction 1. I make a behavioral assumption that despite the weak preference, he will do so.

A1 *Suppose that in round t of auction 1 buyer i believes that $v_i \leq v^{3:N} \leq E[p_2]$, then he bids v_i in round t of auction 1.*

When deciding what to bid in round 1 of auction 1, a bidder makes certain assumptions about what happens till the end of the game. As I have already discussed, in an efficient equilibrium, he assumes that $E[p_2] = E[v^{3:N}]$. Furthermore, I have assumed that he behaves in such a way that A1 is satisfied. Recall also that in an efficient equilibrium, $E[p_1] = E[v^{3:N}]$. This implies that the expected value of the highest bid submitted in round 1 of auction 1 cannot exceed the expected value of the third highest valuation ($E[\max\{b^1_{1,i}(.)\}_{i\in\{1,\ldots,N\}}] \leq E[v^{3:N}]$). From the latter inequality it follows that if bidder i bids at all in round 1 of auction 1, his bid is below the valuation ($b^1_{1,i}(.) < v_i$).

Suppose that in round 1 of auction 1, each bidder i bids $b^1_{1,i}(.) = b(v_i) < v_i$. Since I only focus on monotonic bidding functions, $b(v_i)$ must be strictly increasing. If so, the current price equals the second highest bid ($p^2_1 = b(v^{2:N})$). From the current price bidders infer the second highest valuation ($v^{2:N} = b^{-1}(p^2_1)$). This makes $N-2$ bidders realize that in auction 2, where $E[p_2] = E[v^{3:N}]$, the price will be at least given by their valuations. Then by A1, every bidder i with $v_i < v^{2:N}$ bids v_i in round 2 of auction 1. As a result, in auction 1, the expected final price exceeds $E[\max[v^{3:N}, b(v^{2:N})] > E[v^{3:N}]$. On the other side, there is a Revenue Equivalence Theorem implying that this price should equal $E[v^{3:N}]$. Hence, the described behavior cannot happen in equilibrium. In efficient equilibrium, bidders do not bid early.

To reiterate, early bidding leads to the revelation of the current price which makes weak bidders (those with the valuation lower than $v^{2:N}$) aware that if they are not aggressive, they will not win any good. Therefore, after early bidding the weak bidders start the bidding war (by increasing their bids up to their maximum willingness to pay). This bidding war is undesirable, as it increases the final price. Bidders prefer not to reveal any information in the first place so that the bidding war never starts. All in all, in an efficient equilibrium bidders do not bid early. In appendix, I present an efficient equilibrium in which all the bidders snipe. The following theorem summarizes the result.

Theorem 2.1. *In efficient symmetric perfect Bayesian equilibrium of* Γ_1*, buyers always snipe.*

To conclude, this section presents a model of two consecutive auctions to illustrate multiple Internet auctions with the same offerings. I argue that bidder's behavior in the current auction depends on his expectations about possibility of a positive transaction in the future auction. As long as he is optimistic and believes that he has a chance for a positive transaction in the future auction, he bids relatively low in the current auction. If the current price achieves certain level, he realizes that his valuation is so low that he has to bid his maximal willingness to pay in the current auction to have any chance for a positive transaction. To prevent such a situation from happening, in the current auction, all the bidders prefer to bid so late that after the increase in the current price there is no chance to submit further bids. Then, weak bidders do not bid their maximal willingness to pay in the current auction and the final price is sustained at the reasonable level. If the current price was not revealed, no bidder would ever learn that his valuation is too low to hope for a positive transaction in the future. The immediate implication is that if the current price was hidden from the bidders, sniping would be less attractive.

2.3. Flexible ending rule

2.3.1. The model

This subsection refines the model presented in subsection 2.2.1 to allow for the flexible ending time.

There are again two sequential auctions and $N > 2$ buyers. Each buyer i ($i = 1, ..., N$) has an independent private valuation v_i that satisfies the condition imposed in the subsection 2.2.1. Auctions are run in sequence. After auction 1 ends, auction 2 starts. After auction 2 finishes, the game ends. The utility of buyer i is given as in the subsection 2.2.1.

Each auction a ($a = 1, 2$) has T_a rounds. In each round t ($t = 1, ..., T_a$) of auction a, first, the *current price* is announced; second, bids are collected from the *potential* buyers and third, *active bidders* and the *current winner* are indicated, where the current price, bids, potential buyers, active buyers and current winners are defined as in subsection 2.2.1. Round T_a is determined endogenously. If there are no bids in round 2 of auction a, auction a ends ($T_a = 2$). If there is a bid in round 2, auction a is extended for one more round ($T_a > 2$). The auction continues as long as there is at least one accepted bid. One round without any bids imposes the end of the auction. In other words, if there is a bid accepted in every round $t \in \{2, ..., t^* - 1\}$ and there is no bid accepted in round t^*, then $T_a = t^*$.

Let $h_a^t \in H_a^t$ be a history of the learned outcomes (i.e. current winners, current prices, identity of buyers whose bids equal to current prices, identities of all other active buyers and number of rounds) up to round t of auction a and $s_{a,i}^t \in S$ be a status of buyer i (current winner, losing bidder or bidder who did not bid).in round t of auction a. Then, an action of buyer i in round t of auction a is given by $b_{a,i}^t(v_i, h_a^t, s_{a,i}^t) \in R_+$. Let $\sigma_{a,i}$ be a bidding strategy of bidder i in auction a. I define strategy of bidder i to be $\sigma_i = \begin{bmatrix} \sigma_{1,i} & \sigma_{2,i} \end{bmatrix} \in \Omega_i$. The beliefs are defined as in 2.2.1. $o \in O$, ϖ and Ω have the same meaning as in the subsection 2.2.1. The game is defined by $\Gamma_2 = \{\Omega_1, ...\Omega_N, o(.)\}$.

2.3.2. Results

This subsection studies optimal bidding behavior under a flexible ending time. Here, after bids are accepted, there is at least one more round. Hence, the flexible ending rule destroys all the modeled gains from the late bidding.

Suppose that bids are collected at some round t of auction 1. After bids are accepted, the price is revealed. By symmetry and monotonicity the price is given by the bid sent by the bidder with the second highest valuation. From the price, some bidders infer that in contrast to their opponents they have low valuations (that is lower than $v^{2:N}$). They need to be aggressive to win the good. When they become aggressive, the expected final price increases in auction 1. Strong bidders prefer to hide the information about their valuations from the weak bidders, so that the weak bidders do not become aggressive in auction 1. Under the fixed ending rule, they could do so by sniping. Under the flexible ending rule, the weak bidders always have time to react to the bids of the strong bidders. Then, the strong bidders never want to start informative bidding. As a result, an equilibrium cannot be reached.

Theorem 2.2. *There is no efficient symmetric perfect Bayesian equilibrium of* Γ_2.

Proof. See the Appendix □

To conclude, under the flexible ending time, there is no efficient equilibrium. The previous section showed that under the fixed ending time, there is an efficient equilibrium. I conclude that having the fixed ending time instead of the flexible ending time is better for efficiency.

2.4. Conclusions

More and more transactions take place on Internet auctions. Internet enables trade on a longer distance. It does not require the seller and the potential buyers to be available in the same moment. Therefore, Internet auctions are dynamic and have no activity rules[1]. When having dynamic auctions without activity rules, auction platform needs to decide upon the ending rule. However, they are offered little guidelines from auction theory. That is maybe the reason why they decided for different solutions. eBay chose the fixed ending time, Amazon set the flexible ending time and Yahoo left the decision to the seller. As noted in the introduction of the present chapter, different ending rules trigger different bidding behavior. Under the fixed ending, bidders tend to postpone their bids till the very end of auction. Under the flexible ending time, this is not the case.

The present paper studies the optimal ending rule from the perspective of auction platform which is interested in efficiency. It models sequential dynamic second-price independent private-value auctions under fixed ending time and flexible ending time. In the first auction, buyers wish to bid below their valuations to sustain an option of winning the good in the later auctions. They do not wish to bid early, because early bidding makes some of the bidders aware that they have no chance to win the good in the later auctions and hence, makes them willing to bid their valuations in the first auction, which increases the price in the first auction. Therefore, everyone prefers to bid in the last round of the first auction. Under the fixed ending rule, this is possible. There is an efficient equilibrium with sniping. Under the flexible ending rule, there is no last round. Therefore, there is

[1]See chapter 1 for a discussion on the link between activity rule and ending rule.

no efficient equilibrium. All in all, the fixed ending time is more desirable for efficiency than the flexible ending time.

The present paper leaves space for the future research. Throughout the paper, the major concern was about efficiency. In the absence of a reserve price, an efficient outcome is of natural focus. In reality, Internet auctions might differ with respect to the starting price. It is therefore interesting to check how different starting prices affect the revenue. Other assumption that could be modified relates to the fixed number of bidders. In auction theory, assuming that a bidder knows the number of his opponents is standard. However, it is not obvious that it happens in Internet auctions. It would be interesting to check how uncertainty about the actual size of the market affects bidders' incentives. Finally, the present paper could be empirically tested. It suggests that sniping should be more prevalent in auctions facing fierce competition than those selling less popular goods. It also predicts that sniping should be less prevalent in auctions which do not reveal the current price. Testing these claims would be an interesting empirical contribution.

2.5. Appendix

Before proving theorem 2.1, I need to prove the following proposition.

Proposition 1: *Let $\bar{\sigma}_i$ be defined as follows:*

(1) *in round 2 of auction 1, if $p_2^1 = 0$, buyer i bids $\hat{b}(v_i) = E[v^{3:N}|v^{2:N} = v_i]$,*

(2) *in round 2 of auction 1, if $p_2^1 > 0$, he bids v_i,*

(3) *in round 1 of auction 2, if he is a potential buyer, he bids v_i,*

(4) *otherwise, he does not bid,*

Let $\bar{\mu}_i$ be as follows:

(1) *if $p_2^1 = 0$, buyer does not update his beliefs,*

(2) *if $p_2^1 > 0$, buyer i believes that $v_i < v^{2:N}$,*

(3) *otherwise, he uses Bayes rule,*

Then, $(\bar{\sigma}_1, ..., \bar{\sigma}_N; \bar{\mu}_1, ..., \bar{\mu}_N)$ *is a perfect Bayesian equilibrium (in undominated strategies) of* Γ_1*. The equilibrium outcome is efficient. The expected price equals to* $E[v^{3:N}]$ *in both auctions.*

Proof. Beliefs are constructed so that on-equilibrium, bidders use the Bayes rule. Off-equilibrium, a perfect Bayesian equilibrium does not bind the beliefs. Hence, I can assume that seeing bids in round 1 of auction 1, which happens only off-equilibrium, buyer i believes that $v_i < v^{2:N}$.

Now, I prove that given the beliefs the behavior is sequentially rational. Clearly, in auction 2, it is a dominant strategy to bid one's valuations both on- and off-equilibrium. This is for the same reason that it is a dominant strategy in a single-unit second-price auction. Bidders are indifferent between bidding their valuations in round 1 of auction 2 and round 2 of auction 2. Hence, it is optimal for them to bid their valuations in round 1 of auction 2. Once they do so, they do not wish to bid any longer, as this would require bidding above the valuation, which is a dominated strategy.

Suppose that there were no bids in round 1 of auction 1. Then, the situation becomes equivalent two the first out of the two sequential second-price auctions. Hence, it must be that $E[p_1] = E[p_2] = E[v^{3:N}]$, which implies that $\hat{b}(v_i) = E[v^{3:N}|v^{2:N} = v_i]$ (see Milgrom and Weber, 1982, and Weber,1983 for results on sequential second-price auctions). Note that off-equilibrium, when $p_2^1 = 0$, no information is revealed and hence, it is still optimal to bid $E[v^{3:N}|v^{2:N} = v_i]$ in round 2 of auction 1.

It remains to prove that no-one has an incentive to bid in round 1 of auction 1 and that, if $p_2^1 > 0$, it is optimal for buyer i to bid v_i in round 2 of auction 1. Suppose that buyer j bids in round 1 of auction 1. If he is the only bidder in round 1 of auction 1, the other buyers do not change their behavior and hence, buyer j does not gain anything. If there is at least one other buyer in round 1 of auction 1, the situation of buyer j worsens, as instead of bidding $E[v^{3:N}|v^{2:N} = v_i] < v_i$ his opponents will bid their valuations in round 2 of auction 1. Hence, he has no incentives to bid in round 1 of auction 1. Finally, note that if $p_2^1 > 0$, buyer i believes that $v_i < v^{2:N}$. Since all the potential buyers bid their valuations in auction 2, he will not win the good at a profitable price in auction 2. Hence, he weakly prefers to bid v_i in round 2 of auction 1. □

Theorem 2.1: *In efficient symmetric perfect Bayesian equilibrium of* Γ_1*, in auction 1, buyers always snipe.*

Proof. Proposition 1 implies that there exists an efficient equilibrium of Γ_1 in which bidders wait with bidding for round 2. Hence, it is sufficient to prove that there does not exist an efficient symmetric perfect Bayesian equilibrium in which every bidder i bids in round 1 of auction 1. Suppose otherwise. By the Revenue Equivalence Theorem, in efficient Bayesian Nash equilibrium, $E[p_2] = E[v^{3:N}]$ (see Krishna, 2002). Let each buyer i bid $\bar{b}(v_i) \geq 0$ in round 1 of auction 1. By monotonicity, $p_1^2 = \bar{b}(v^{2:N})$. Since p_1^2 is revealed, all the bidders learn $v^{2:N}$. Since identity of the current winner is indicated, everyone knows who has the highest valuation. By A1, every bidder i with $v_i < v^{2:N}$ bids v_i in round 2 of auction 1. As a result, $E[p_1] \geq E[\max[\bar{b}(v^{2:N}), v^{3:N}]]$. Since there is always a positive probability that $\bar{b}(v^{2:N}) > v^{3:N}$, $E[\max[\bar{b}(v^{2:N}), v^{3:N}]] > E[v^{3:N}]$ and thus $E[p_1] > E[v^{3:N}]$. But the Revenue Equivalence Theorem implies that $E[p_1] = E[v^{3:N}]$ in an efficient Bayesian Nash equilibrium (see Krishna, 2002). Hence, the described strategy profile is not a Bayesian Nash equilibrium and as a consequence not a perfect Bayesian equilibrium. A contradiction. □

Theorem 2.2: *There is no efficient symmetric perfect Bayesian equilibrium of* Γ_2*.*

Proof. Note that by the Revenue Equivalence Theorem, in an efficient Bayesian Nash equilibrium, $E[p_2] = E[v^{3:N}]$ (see Krishna, 2002). First, suppose that in round 1 of auction 1, each bidder i bids $b(v_i)$, st. $b(0) = 0$ and $b'(v_i) > 0$. Then, $p_1^2 = b(v^{2:N})$ and $v^{2:N}$ is revealed. By A1 every bidder i with $v_i < v^{2:N}$ bids v_i in round $t+1$ of auction 1. Hence, $E[p^1] \geq E[\max[b(v^{2:N}), v^{3:N}]] > E[v^{3:N}]$, which by the Revenue Equivalence Theorem is impossible in an efficient Bayesian equilibrium in which bidder with $v_i = 0$ obtains zero utility.

Second, suppose that there is no bid submitted in round 1 of auction 1 and that in round 2 of auction 1, each bidder i bids $b(v_i)$, st. $b(0) = 0$ and $b'(v_i) > 0$. Then, $p_1^3 = b(v^{2:N})$ and $v^{2:N}$ is revealed. By A1 every bidder i with $v_i < v^{2:N}$ bids v_i in round $t+1$ of auction 1. Hence, $E[p^1] \geq E[\max[b(v^{2:N}), v^{3:N}]] > E[v^{3:N}]$, which by the Revenue Equivalence Theorem is impossible in an efficient Bayesian equilibrium in which bidder with $v_i = 0$ obtains zero utility.

Hence, in a symmetric efficient Bayesian equilibrium, there are no bids submitted in rounds 1 and 2 of auction 1 and auction 1 ends without any bids. In auction 2, every potential bidder weakly prefers to bid the valuation. Hence, the expected utility of bidder i is:

$$\int_0^{v_i} (v_i - v_j) dF^{1:N-1}(v_j) = (N-1) \int_0^{v_i} \int_0^{v_2} (v_i - v_2) dF^{1:N-2}(v_3) dF(v_2) \tag{2.1}$$

Consider a deviation of bidder i by bidding v_i in round 2 of auction 1. After his deviation, his opponents will bid at most their valuations in auction 1 and in auction 2. Then, in the worst scenario, in auction 1, the highest bid of his opponents will be given by $v^{1:N-1}$ and in auction 2, the highest bid of his opponents will equal to $v^{2:N-1}$. His expected utility of bidder i will then amount to:

(2.2)
$$(N-1) \int_0^{v_i} \int_0^{v_2} (v_i - v_2) dF^{1:N-2}(v_3) dF(v_2) + (N-1) \int_{v_i}^{1} \int_0^{v_i} (v_i - v_2) dF^{1:N-2}(v_3) dF(v_2)$$

where the first term describes the situation in which he wins the good in auction 1 and the second term refers to the situation in which he wins the good in auction 2. Clearly, (2.1) is smaller than (2.2). Hence, the deviation is profitable, which contradicts the claim that in a Bayesian equilibrium, there are no bids submitted in rounds 1 and 2 of auction 1 and auction 1 ends without any bids. I have already proven that there is no efficient perfect Bayesian equilibrium in which bidders bid in round 1 of auction 1 or round 2 of auction 1. Hence, there is no symmetric efficient perfect Bayesian equilibrium. □

CHAPTER 3

Coordination Failure in Simultaneous Internet Auctions

3.1. Introduction

eBay is a platform allowing sellers to run auctions and buyers to submit their bids. Each auction is scheduled for a fixed period, so that on the one side, a bidder has plenty of time to submit a bid and on the other side, he is able to snipe, that is to submit a bid in the closing seconds of the auction. The same goods are offered for sale in different auctions. Since auctions are dynamic, it happens that the same goods are auctioned in simultaneous auctions and that there are several auctions ending in the same point of time. The present chapter asks a question whether such situation is desirable for the seller and what the optimal behavior of the bidders is.

Internet auctions have been widely studied in the literature (see Bajari and Hortacsu 2004 for an overview of the literature on Internet auctions). Most of the models study single-unit auctions, ignoring possible effects of multiple offerings. Peters and Severinov (forthcoming) focus on simultaneous Internet auctions. In their model, there is no strict ending rule, which is the reason why they find an efficient equilibrium in simultaneous Internet auctions.

The present paper studies two simultaneous Internet auctions, which simultaneously start and simultaneously end. Buyers have unit demand and wish to coordinate between the auctions, so that no-one pays too high a price. On the other hand, they wish to postpone bidding to hide the information from the opponents. The willingness to wait with bidding determines equilibrium. Bidders end up bidding so late, that they do not manage to efficiently split between the competing auctions. This rationalizes a phenomenon of sniping, commonly observed in online auctions and suggests that simultaneous Internet auctions are bad for efficiency.

The paper is structured as follows. The next section presents the model of dynamic simultaneous auctions. Section 3.3 eliminates equilibrium candidates in which all the buyers bid early. Section 3.4 presents equilibrium with sniping, which is unique. Section 3.5 concludes.

3.2. The model

There are two auctions and $N > 3$ buyers. Each auction a $(a = 1, 2)$ offers the same indivisible good for sale. Each buyer i $(i = 1, ..., N)$ has an independent private valuation of one (and only one) item of the good (v_i), which is distributed according to distribution $F(v_i)$ with density $f(v_i)$ and support on $[0, 1]$. $v^{m:n}$ denotes the m^{th} highest valuation out of n bidders. $F^{m:n}(.)$ is distribution of $v^{m:n}$. $f^{m:n}(.)$ is corresponding density.

The games takes two stages[1]. In each stage t $(t = 1, 2)$ in auction a the following happens:

- The current price (p^t_a) is derived.
- Buyers submit bids.
- The current winner is identified.

After learning the current price (p^t_a), each buyer i may submit a bid in auction a $(b^t_{a,i} > 0)$ or not to bid at all $(b^t_{a,i} = 0)$. I assume that only one bid per stage is allowed (that is $b^t_{1,i} = 0$ or $b^t_{2,i} = 0$). I focus only on monotonic strategies (that is $\frac{\partial}{\partial v_i} b^t_{a,i} > 0$ or $b^t_{a,i} = 0$).

At t, buyer i becomes an *active bidder* in auction a, if he overbids the *current price* in auction a (p^t_a) and his previous bids submitted in auction a (i.e. if $b^t_{a,i} > \max[p^t_a, \max\{b^k_{a,i}\}_{k<t}]$). He becomes a *current winner* in auction a, if (1) by t he submits the highest bid in auction a as the first one or (2) he is randomly chosen from the set of buyers who have simultaneously submitted the highest bid in auction a by t. Hence, for each auction a, he can have three statuses: an inactive bidder, an active losing bidder and a current winner.

[1]If there were more stages, efficient equilibrium would be still impossible to reach.

At the beginning of each stage t, the current prices are communicated to the bidders. In stage 1, the current price equals zero in both auctions. In round 2 of auction a, the current price remains zero, as long as the number of active bidders does not exceed one. If there are at least two active bidders in round 1 of auction a, the current price (p_a^2) equals to the highest bid submitted by the active losing bidder.

After the last stage, final winners and final prices are identified. The current winner of round 2 of auction a becomes the final winner. The final price of auction a is chosen on the same basis as the current price.

After the last stage, the utility of buyer i is given by:

$$u_i(.) = \begin{cases} v_i - p_1 - p_2 & \text{if } i \text{ is the final winner in both auctions} \\ v_i - p_a & \text{if } i \text{ is the final winner only in auction } a \\ 0 & \text{otherwise} \end{cases}$$

Let $h_i^t \in H_i^t$ be a history of the learned outcomes (i.e. current winners, current prices, identity of buyers whose bids equal to current prices and identities of all other active buyers) and statuses of buyer i in each auction a up to stage t ($t = 1, 2$). Then, an action of buyer i in stage t is given by $\sigma_i^t = (b_{1,i}^t(v_i, h_i^t), b_{2,i}^t(v_i, h_i^t))$. The strategy of buyer i is described by the function:

$$\sigma_i = \left[\begin{array}{cc} \sigma_i^1(v_i, h_i^t) & \sigma_i^2(v_i, h_i^t) \end{array} \right] : [0,1] \times H_i^1 \times H_i^2 \rightarrow R_+^4$$

Ω_i is the space of all possible σ_i and $\Omega_i^t(.)$ is the space of all possible $\sigma_i^t(.)$. Let μ_i be a system of beliefs of buyer i on the valuations of the opponents. The prior belief is that each valuation is distributed according to $F(.)$. Let $\sigma = (\sigma_i, ..., \sigma_N)$ and $\mu = (\mu_1, ..., \mu_N)$. I adopt standard notation of writing $\sigma_{-i} = (\sigma_1, ..., \sigma_{i-1}, \sigma_{i+1}, ..., \sigma_N)$ and $\mu_{-i} = (\mu_1, ..., \mu_{i-1}, \mu_{i+1}, ..., \mu_N)$. The final outcome, including final prices and final winners for both auctions, is denoted by $o \in O$. The outcome function is $\varpi : \Omega \times [0,1]^N \rightarrow O$, where $\Omega = \Omega_1 \times \cdots \times \Omega_N$. The game is denoted by $\Gamma^1 = \{\Omega_1, ...\Omega_N, o(.)\}$. To solve the game I impose the following assumptions on strategies.

A1: *Let $u_i^t(\hat{\sigma}_i^t(.)|\sigma,\mu)$ be the expected utility over the final outcome of buyer i who in stage t plays $\hat{\sigma}_i^t(.)$, given h_i^t, μ and σ. If $u_i^t((0,.)|\sigma,\mu) \geq u_i^t(\sigma_i^t(.)|\sigma_{-i})$, then $\sigma_i^t(.) = (0,.)$. If $u_i^t((.,0)|\sigma,\mu) \geq u_i^t(\sigma_i^t(.)|\sigma_{-i})$, then $\sigma_i^t(.) = (.,0)$.*

A2: *At least two buyers follow the same strategies.*

A3: *If for some value v_j bidder $j \in \{1,...,N\}$ is supposed to submit a bid $b(v_i) > 0$ in auction a in round t, then every buyer $i \neq j$ with $v_i = v_j$ is also supposed to submit bid $b(v_j)$ in auction a in round t.*

A1 excludes all unnecessary bids (i.e. the ones that have no effect on the expected payoff) and could be easily justified by cost of bidding. A2 and A3 assure the minimal level of symmetry.

3.3. Early bidding

The game has two interesting strategies: early bidding and sniping. The early bidding is meant as bidding at stage 1. Sniping is defined as not bidding at stage 1 and bidding at stage 2. To prove rationality of sniping, it is required to show that bidders do not bid early.

Let each buyer i bid $b(v_i)$ (satisfying $b'(v_i) > 0$) in auction 1 at stage 1 (that is $b_{1i}^1 = b(v_i)$ for every i). The presence of two objects on sale implies the expected market price of $E[v^{3:N}]$. In order not to pay more, bidders start with low bids. Therefore, $b(v_i) < v_i$.

Suppose now that after each buyer i bids $b(v_i) < v_i$, buyer 1 becomes the current winner and the current price in auction 1 is given by $p_1^2 = b(v_2)$. From such an outcome, every buyer learns that buyer 1 has the highest valuation ($v_1 = v^{1:N}$) and buyer 2 has the second highest valuation ($v_2 = v^{2:N}$). Note that there is always positive probability that $b(v_1) > v_2$, so that buyer 2 has no chance for a positive transaction in auction 1. Therefore, to assure efficiency, buyer 2 needs to win the good in auction 2. Then, no buyer $i \in \{3,...,N\}$ has a chance to win the good in auction 2 at a reasonable price. Hence, he will try to win the good in auction 1. He will have a chance to win the good in auction 1 at a price below v_i, as long as buyer 1 does not increase his bid in auction 1. If buyer 1 bids above v_2 in auction 1 in stage 2, no buyer $i \in \{3, ..., N\}$ has a chance to make a positive transaction in auction 1. Not being able to win the good at a reasonable price, buyer $i \in \{3, ..., N\}$ is not interested in bidding in auction 1 in stage 2. But when he is not interested, buyer 1 is also not interested in bidding in auction 1 in stage 2. Hence, a pure equilibrium cannot be reached.

In other words, buyer with the highest valuation is willing to bid below the valuation in auction 1 in stage 1 to avoid the situation in which he ends up paying $v^{2:N}$. When he does so, he sends an open invitation for weak buyers to bid in auction 1 in stage 2. He must then bid aggressively in auction 1 in stage 2 to discourage weak buyers from bidding in auction 1 in stage 2. Such aggressive behavior does not affect the final outcome and hence is excluded by A1. The coordination is impossible in the equilibrium.

Lemma 3.1. *There does not exist an efficient perfect Bayesian equilibrium in which all the buyers bid in the same auction in stage 1.*

Proof. See appendix. □

It is now natural to ask whether all the buyers bid in the first stage but in different auctions. Suppose $m > 1$ buyers bid $\bar{b}(v_i) < v_i$ (satisfying $\bar{b}'(v_i) > 0$) in auction 1 in stage 1 and $N - m$ bid $\hat{b}(v_i) < v_i$ (satisfying $\hat{b}'(v_i) > 0$) in auction 2 in stage 2. Suppose also that buyer 1 is a current winner in auction 1 in stage 1, buyer N is a current is a current winner in auction 2, $p_1^1 = \bar{b}(v_2)$, $p_2^1 = \hat{b}(v_3)$ and $\bar{b}^{-1}(v_2) > \hat{b}^{-1}(v_3)$. Then, everyone learns v_2 and knows that $v_2 > v_j$ for each $j \in \{3, ..., N-1\}$. Suppose in auction 2 in stage 2, buyer 2 bids v_2 and buyer N bids v_N. Then, each buyer $j \in \{3, ..., N-1\}$ has no chance for a profitable transaction in auction 2. If so, he will consider bidding in auction 1. As long as buyer 1 does not bid above v_2 in auction 1 in stage 2, there is a positive probability that buyer j will win the good in auction 1 at a price below v_j. Otherwise, he has no reason to bid in auction 1 in stage 2. Since buyer 1 is interested in bidding only when other buyers bid in auction 1 in stage 2, a pure equilibrium cannot be reached.

In other words, buyer with the highest valuation aims at an efficient allocation with the expected final price of $v^{3:N}$ in both auctions. To coordinate for such an allocation, he needs to first, bid low and second, to send an unnecessary bid. Since I exclude all the unnecessary bids from the model, I can conclude that there is no efficient equilibrium in which all the buyers bid early.

Proposition 3.2. *There does not exist an efficient perfect Bayesian equilibrium in which in stage 1, $m < N$ buyers bid in auction 1 and $N - m$ buyers bid in auction 2.*

Proof. See appendix. □

3.4. Sniping

As already shown, there is no efficient equilibrium with early bidding. It remains to check whether there is an equilibrium with sniping.

Suppose no buyer i bids at stage 1. Then, it is optimal for each buyer i to bid the valuation in the optimally chosen auction at stage 2. Buyers divide between the auctions, so that the difference between the number of active bidders in auction 1 and the number of active bidders in auction 2 is smaller or equal to 1. This guarantees that no buyer is willing to reallocate.

Theorem 3.3. *In a perfect Bayesian equilibrium, each buyer i does not bid in stage 1 and bids v_i in stage 2. m buyers (where m is $\frac{N}{2}$, when N is even and $\frac{N+1}{2}$ when N is odd) bid in one auction and $N - m$ buyers bid in the other auction. The resulting outcome is inefficient.*

Proof. See Appendix. □

To conclude, although the auction is dynamic, bidders do not make use of the time to efficiently allocate between the auctions. There is an equilibrium in which buyers wait with bidding for the last stage. As a result, no information is revealed. Having no information, buyers are unable to coordinate between the auctions and the resulting outcome is inefficient.

3.5. Conclusions

The present paper demonstrates a stylized model of simultaneous Internet auctions. Buyers want to optimally coordinate between the competing auctions. To do so, they need to learn the information from the market. The revealed information does not only enable a desired allocation, but also makes weak bidders aggressive. To avoid bidding war, the strong bidder needs to bear unnecessary cost of sending an irrelevant bid. Since he is unable to commit to bear it, the coordination is impossible. As a result, the phenomenon of sniping takes place. The final allocation is inefficient, which implies that simultaneous auctions are bad for efficiency.

3.6. Appendix

Proof. (lemma 3.1) Without loss of generality suppose that each buyer $i \in \{1, ..., N\}$ bids $b^1_{1,i}(.) = b(v_i)$ (st. $b'(v_i) > 0$) in auction 1 in stage 1 and that buyer 1 is a current winner of auction 1 in stage 1 and $p^2_1 = b(v_2)$. Then, it is revealed that buyer 1 has the highest valuation ($v_1 = v^{1:N}$) and that buyer 2 has the second highest valuation ($v_2 = v^{2:N}$). Furthermore, from the price buyers learn v_2. Suppose there is an efficient equilibrium. Then, by the Revenue Equivalence Theorem the expected final price of auction 1 is given by $E[v^{3:N}]$. This implies that $b(v_i) < v_i$.

We now derive the optimal bidding behavior in stage 2. To have an efficient allocation, buyer 1 or buyer 2 has to bid in auction 2. Suppose that buyer 2 bids there. In order to discourage others from bidding in auction 2, buyer 2 bids the valuation in auction 2. Then, no-other buyer has interest in bidding in auction 2. If buyer 1 bids above v_2 in auction 1, no-one else bids in auction 1 by A1. If no buyer $i \neq 1$ bids in auction 1 in stage 2, buyer 1 does not bid in stage 2 by A1. But when he does not bid, there is a positive probability that $b^1_{a,i}(v^{1:N}, .) < v_i$ for some $v_i < v^{2:N}$ and hence there is a positive probability that buyer with $v_i < v^{2:N}$ has incentives to bid v_i in auction 1 in stage 2. All in all, a pure equilibrium cannot be reached.

Suppose now buyer 1 bids in auction 2 in stage 2. In order to discourage others from bidding in auction 2, buyer 1 bids at least v_2 in auction 2. Then, no-one else has interest to bid in auction 2. Buyer 2 prefers to bid v_2 in auction 1 in round 2. Then, in auction 1, the expected price is at least given by $E[\min[b(v_1), v_2]] > E[v_3]$, which by the Revenue Equivalence Theorem is impossible in efficient equilibrium in which bidder with $v_i = 0$ obtains zero utility. □

Proof. (proposition 3.2) Without loss of generality suppose that each buyer $i \in \{1, ..., m\}$, where $m > 2$, bids $b_{1,i}^1(.) = \bar{b}(v_i)$ in auction 1 in stage 1 and that each buyer $i \in \{m+1, ..., N\}$ bids $b_{2,i}^1(.) = \hat{b}(v_i)$ in auction 2 in stage 1. By A2, $m < N - 1$. Suppose also that buyer 1 is a current winner of auction 1 in stage 1, buyer N is a current winner of auction 2 in stage 2, $p_1^2 = \bar{b}(v_2)$ and $v_2 > v_j$ for all $j \in \{1, ..., N\}/\{1, 2, N\}$. Then, it is revealed that buyer 1 or buyer N has the highest valuation and that buyer 2 or buyer N has the second highest valuation. Suppose there is an efficient equilibrium. Then, by the Revenue Equivalence Theorem the expected final price of auction 1 is given by $E[v^{3:N}]$. This implies that $\bar{b}(v_i) < v_i$.

We first derive optimal behavior in auction 2 in stage 2. Since buyer 2 has no chance to profitably win against buyer 1 and has a positive chance of winning in auction 2, he bids v_2 in auction 2 in stage 2. In reply to this, buyer N bids v_N in auction 2 in stage 2, provided that $\hat{b}(v_N) < v_2$ and $v_2 < v_N$. Otherwise, by A1 he does not bid. By A1 other buyers do not bid in auction 2 in stage 2.

We now derive optimal bidding behavior in auction 1. Buyer 1 has incentives to bid in auction 1 in stage 2, only if other buyer bids j in auction 1 in stage 2. But buyer j has incentives to bid in auction 1 in stage 2, only if buyer 1 does not bid in auction 1 in stage 2. All in all, a pure equilibrium cannot be reached. □

Proof. (theorem 3.3) Lemma 3.1 and proposition 3.2 imply that there is no equilibrium with bidding at stage 1. If there is no bidding at stage 1, it is optimal for each buyer i to bid v_i in stage 2. It must be that m buyers bid in one auction and $N - m$ buyers bid in the other auction, as otherwise there would be a buyer willing to reallocate. The resulting outcome is inefficient, as it can happen that buyer with $v^{1:N}$ and buyer with $v^{2:N}$ bid in the same auction. The deviation is unprofitable, as a single bid does not affect the current price and hence, does not reveal any information. Not having new information, bidders bid in the way that the payoff of the deviating bidder remains unaffected. $\square$

CHAPTER 4

Multiple and Late Bidding in Internet Auctions

4.1. Introduction

Imagine a seller who has several units of the same good for sale (e.g. several new memory cards). He wishes to sell each unit separately using Internet auctions such as that on eBay. He can decide for simultaneous, overlapping or sequential auctions.

Existing studies on Internet auctions mainly focus on single-unit auctions and hence, have little to say whether the seller should schedule simultaneous, sequential or overlapping auctions and how bidders should behave in the presence of multiple auctions offering identical goods (see Lucking-Reiley (2000) for an overview of the existing Internet auctions and Bajari and Hortacsu (2004) for an overview of the existing studies on Internet auctions). Peters and Severinov (forthcoming) study simultaneous Internet auctions without fixed ending time, which is not only one of the key characteristics of auctions on eBay, but also an important factor affecting strategic interactions. The empirical studies indicate the popularity of sniping, which is placing a bid in the closing seconds of an auction, and multiple bidding in Internet auctions on eBay (see Bajari and Hortacsu (2003), Bajari and Hortacsu (2004), Roth and Ockenfels (2002)).

This paper focuses on the simultaneous and overlapping independent private value Internet auctions with a fixed ending time. I provide a rational for the two empirically observed strategies: late bidding and multiple bidding. I also argue that in contrast to overlapping Internet auctions, simultaneous Internet auctions might be bad for efficiency.

In present a model with a risky bid transmission in the last minute, which was previously modeled by Roth and Ockenfels (2006). The strategic interactions between bidders are similar to the ones illustrated by Peters and Severinov (forthcoming), who model competing Internet auctions without fixed ends. Bidders are willing to coordinate between the auctions so that no-one pays too much. Because of the presence of the fixed end time, it is not any longer profitable for them to use the algorithm proposed by Peters and Severinov. The coordination is achieved by early bidding which reveals some information on the ordering of their valuations. After learning the (partial) ordering of the valuations, the strongest bidders bid in the opposite auctions. Then, the prices are sustained at a reasonable level. When bidders learn the (partial) ordering of their valuations early, they have time to safely reallocate before the last minute and the resulting outcome is efficient. When the reallocation takes place in the risky last-minute, the resulting outcome might be inefficient. Foreseeing that their bids might be rejected in the last minute, bidders bid more aggressively when testing the ordering of their valuations. As a result, the expected price in one auction increases as compared to the expected price in an efficient equilibrium. In the other auction, the price decreases, as it depends on the bid which is submitted in the risky last minute. This situation happens only in simultaneous auctions. When auctions are overlapping, there is always time to safely reallocate. The resulting outcome is always efficient.

The paper is structured as follows. Section 4.2 studies simultaneous auctions. Subsection 4.2.1 introduces the model of simultaneous Internet auctions. Subsection 4.2.2 establishes possible efficient equilibria, each basing on the multiple bidding. Subsection 4.2.3 presents possible inefficient equilibria, that incorporate last-minute bidding. Subsection 4.2.4 summarizes the possible equilibrium outcomes of the model of simultaneous Internet auctions. Section 4.3 extends the model to allow for overlapping Internet auctions and discusses the equilibria of the extended model. Finally, section 4.4 concludes.

4.2. Simultaneous Internet auctions

This paper focuses on the situation in which the seller has two units for sale and can choose between simultaneous and overlapping Internet auctions. The situation is as simple as possible. There is no competition from the outside world. The reserve prices are fixed at zero. The set of bidders is fixed. The two schedules impose similar incentives for the bidders. Bidders are willing to learn some information on the ordering of their valuations, so that they will be able to appropriately reallocate between the auctions. Yet, the schedule of the auctions affects the final allocation and the revenue. This section studies what happens if auctions are run simultaneously.

4.2.1. The model

This subsection presents a simple model of simultaneous Internet auctions. There are two second-price auctions selling one item of the same good (auction 1 and auction 2) and 3 risk-neutral buyers[1]. Buyer i ($i = 1, 2, 3$) has an independent private valuation of one (and only one) item of the good (v_i), which is distributed according to distribution $F(v_i)$ with density $f(v_i)$ and support on $[0, 1]$. $v^{m:n}$ denotes the m^{th} highest valuation out of n bidders. $F^{m:n}(.)$ is distribution of $v^{m:n}$. $f^{m:n}(.)$ is corresponding density.

There are three stages[2]. At every stage t ($t = 1, 2, 3$) the following happens:

(1) Each auction $a \in \{1, 2\}$ announces the *current standing price* (p^t_a).

(2) Each bidder i submits a bid $b^i_{at} \geq 0$.

(3) Each auction a indicates all *active bidders* and chooses the *current winner.*

A bidder is allowed to submit only one bid per round[3]. At $t < 3$, he becomes an *active bidder* in a given auction, if he overbids the current standing price and his previous bids submitted in this auction (i.e. i is an active bidder in auction a at $t < 3$, if $b^i_{at} \geq \max[p_{at}, \max_{\tilde{t}<t} b^i_{a\tilde{t}}]$). At $t = 3$, he becomes an active bidder under the same condition with probability $\alpha \in (0, 1)$. α captures possible uncertainty which might occur when a bidder bids in the last minute. It was originally proposed by Roth and Ockenfels (2006).

Player i becomes a *current winner* in auction a, if (1) he submits the highest bid in auction a as the first one or (2) he is randomly chosen from the set of players who have

[1]Subsection 2.4 discusses how a larger number of bidders and larger number of auctions affects identified equilibria.

[2]The number of periods is chosen so that all the interesting trade-offs are incorporated and the picture does not become too messy. All the equilibria could be found for any finite number of periods (see subsection 2.4 for details).

[3]Bidding simultaneously in several Internet auctions is rather technically impossible.

simultaneously submitted the highest bid in auction a. The *current standing price* equals the highest bid submitted by the player who is not the current winner. If there is only one player, the current standing price is zero. After the last stage a current winner in auction a becomes a final winner. He wins an object and pays the final price (p_a), which is chosen on the same basis as the current standing price. The utility of the player who is awarded two objects is given by: $u_i(.) = v_i - p_1 - p_2$. If player i is only a final winner in one auction $a \in \{1, 2\}$, his utility equals to $u_i(.) = v_i - p_a$. Finally, a player who does not win any object receives a zero payoff.

For any history $h_{it} \in H_{it}$ an action of player i at time t is given by $X_{it} = \{b^i_{at}(h_{it})\}_{a\in\{1,2\}}$. The strategy of the player i is described by the function

$$\sigma_i = \begin{bmatrix} \sigma_{i1}(h_{i1}) & \sigma_{i2}(h_{i2}) & \sigma_{i3}(h_{i3}) \end{bmatrix} : H_{i1} \times H_{i2} \times H_{i3} \rightarrow X_{i1} \times X_{i2} \times X_{i3}$$

which for every information set indicates the value of the bid that player i sends in each auction at every stage $t = 1, 2, 3$. The final outcome is given by $h_{T+1} \in H_{T+1}$. I define an outcome function $o : \Omega \rightarrow H_{T+1}$, where $\Omega = \Omega_1 \times \Omega_2 \times \Omega_3$.The game is denoted by $\Gamma^1 = \{\Omega_1, \Omega_2, \Omega_3, o(.)\}$.

To conclude, there are two identical objects for sale. The number of rounds is finite. At the last round transmission of a bid is uncertain. The final price is determined by the value of the highest bid submitted by a bidder who is not a current winner in this auction.

4.2.2. Efficient equilibria

When there are two auctions, bidders do not want to bid their valuations in one of the available auctions without learning some information about the opponents. Suppose they do so. Let each buyer bid in round 1 of auction 1. Then the buyer with the highest valuation becomes the current winner and the current price is $v^{2:3}$ in auction 1. The other buyers have no chance to win the good in auction 1 and hence, they move to auction 2. Auction 2 is their last chance to win the good, so they weakly prefer to bid their valuations. The buyer with the second highest valuation becomes the current winner an the current price is $v^{3:3}$. The buyer with the lowest valuation then have no chance for a positive transactions in both auctions, so he does not bid any longer. The buyer with the highest valuation wins the good for the price of $v^{2:3}$ and the buyer with the second highest valuation wins the good for the price of $v^{3:3}$. This is not incentive compatible. The buyer i prefers to bid zero in auction 1 and to bid the valuation in auction 2. Then, if his valuation is higher than $v^{3:3}$, he always wins the good at the price of $v^{3:3}$. Otherwise, his utility is zero. This outcome guarantees the same (efficient) allocation and lower prices and hence, is more attractive. To have an efficient equilibrium, one needs specify bidding so that a bidder is indifferent between wining the good in auction 1 and auction 2. In other words, the expected prices have to be equal in the two auctions.

In an equilibrium, buyers learn the (partial) ordering of their valuations and use this information to appropriately reallocate between the auctions. This way, they assure that player with the highest valuation and player with the second highest valuation do not fight against each other in one auction. Consider first a symmetric equilibrium. Let each buyer i bid $b(v_i)$, st. $b'(v_i) > 0$, in round 1 of auction 1. Then, at round 2 of auction 1, the current price is $b(v^{2:3})$. Since $b'(.) > 0$, bidders learn $v^{2:3}$. They also know that the current winner must have the highest valuation. In auction 2, the current price is 0. The losing bidders find it attractive to reallocate to auction 2. The winning bidder wishes to assure that no-one will fight against him in auction 1. Therefore, he bids the valuation in auction 1. Then, his opponents have no chance for a positive transaction in auction 1. Therefore, they bid their valuations in auction 2. The resulting outcome is efficient. The price is given by $b(v^{2:3})$ in auction 1 and by $v^{3:3}$ in auction 2. The two prices must equal in expectation, that is $E[b(v^{2:3})] = E[v^{3:3}]$. The equality is satisfied for:

$$b(v_i) = v_i - \frac{1}{F(v_i)} \int_0^{v_i} F(v_3) dv_3 \tag{4.1}$$

Proposition 4.1 presents the corresponding equilibrium.

Proposition 4.1. *A strategy profile* $(\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3)$*, where* $\hat{\sigma}_i$ *is defined as follows:*

(1) *at* $t = 1$*, bid according to (4.1) in auction* 1*,*

(2) *at* $t = 2$*, if you win, bid* v_i *in auction* 1*,*

(3) *at* $t = 2$*, if you don't win, bid* v_i *in auction* 2*,*

(4) *otherwise, do not bid.*

is a Bayesian Nash equilibrium of Γ^1*. The equilibrium outcome is efficient. The expected price equals to* $E[v^{3:N}]$ *in both auctions.*

Proof. See Appendix. □

Proposition 4.1 presents an equilibrium in which all the bidders bid according to the monotonic bidding function in auction 1. After doing so, they learn the ordering of their valuations. Having this information, they know how to allocate between the auctions, so that the resulting outcome is efficient. As I will argue now, bidders do not need to first bid in one auction to achieve an efficient allocation.

Suppose bidder 3 bids v_3 in auction 2. Then, buyer 1 and buyer 2 prefer to bid in the auction 1, so that they do not end up paying too much. They do not wish to bid their valuations in auction 1, as one of them might end up paying $v^{2:3}$, which is too much. They prefer to use the strictly increasing bidding function $(b(v_i))$ which for each valuation assigns the bid below the valuation. Once they do so, the one who has the highest valuation becomes the current winner. The current price is $b(\min[v_1, v_2])$. The losing buyer infers that he has no chance to win against the winning buyer. He prefers to bid the valuation in auction 2. If his valuation is higher than v_3, he wins the good at the price equal to $v^{3:3}$. Otherwise, buyer 3 wins the good at the price of $v^{3:3}$. The allocation is efficient and the final prices are given by: $b(\min[v_1, v_2])$ and $v^{3:3}$. The two prices must again equal in the expectation. That is: $E[b(\min[v_1, v_2])] = E[v^{3:3}]$, which is satisfied if:

$$b(v_i) = v_i - \int_0^{v_i} F(v_j)dv_j \tag{4.2}$$

The next proposition specifies the exact strategies of an asymmetric efficient equilibrium.

Proposition 4.2. *Define $\tilde{\sigma}_i$ as follows:*

(1) *at $t = 1$, bid according to (4.2) in auction 1,*

(2) *at $t = 2$, bid v_i*

  (a) *in auction 1, if you have not lost in auction 1 and have not won in auction 2,*

  (b) *in auction 2, otherwise,*

(3) *at $t = 3$, do not bid,*

and $\bar{\sigma}_3$ as follows:

(1) *at $t = 2$ bid v_3 in auction 2,*

(2) *at $t \neq 2$ do not bid.*

Then, $(\tilde{\sigma}_1, \tilde{\sigma}_2, \bar{\sigma}_3)$ is a Bayesian Nash equilibrium of Γ^1. The equilibrium outcome is efficient. The expected price equal to $E[v^{3:3}]$ in both auctions.

Proof. See appendix. □

Propositions 4.1 and 4.2 present two types of efficient equilibria. In one of them, all the three buyers test the ordering of their valuations and later appropriately reallocate. In the other, only two bidders test the order of their valuations and later appropriately reallocate. The allocation is efficient, because the reallocation takes place before the risky last stage. The immediate implication of the efficient allocation is the equivalence of the expected prices in the two auctions. By changing identities of the auctions and bidders, adding some irrelevant bids and shifting bids in time, one can construct other efficient equilibria. The summary of the possible equilibrium outcomes will be presented in the subsection 4.2.4.

4.2.3. Inefficient equilibria

The previous subsection argued that it is not rational to bid one's valuations, without having some information on the valuations of the opponents. It is more reasonable to learn a bit about the ordering of the valuations, so that the final choice of the auction is optimal.

In the presented equilibria, after learning the (partial) ordering of the valuations, bidders reallocated. The reallocation took place before the risky last stage. This subsection shows what happens if bidders reallocate in the risky last stage.

Suppose first that all the bidders bid $b(v_i)$, st. $b'(v_i) > 0$, in auction 1 in round 2. In round 3, the losing bidders reallocate to auction 2. Because of the risky bid transmission in the last minute, some of their bids might be rejected. Foreseeing it, buyers wish to bid more aggressively in auction 1 as compared to what they bid in auction 1 in an efficient symmetric equilibrium presented in the previous subsection. As a result, the expected price increases in auction 1 ($E[p_1] = E[b(v^{2:3})] > E[v^{3:3}]$). In the same time, because of the risky bid transmission in the last stage, the expected price decreases in auction 2 ($E[p_2] = \alpha E[v^{3:3}] < E[v^{3:3}]$). But then, buyer i finds it more attractive to bid the valuation in the auction 2 in round 2. This way, he assures that his bid is never rejected and he pays the lowest possible price. As a result, there is no symmetric equilibrium with last-minute bidding (the formal argument is presented in lemma 4.8 in the appendix).

Suppose now that only two bidders (buyer 1 and buyer 2) bid $b(v_i)$, st. $b'(v_i) > 0$, in auction 1 in round 2 and later, the losing bidder bids the valuation in the auction 2 and the winning bidder bids the valuation in the auction 1. Then, buyer 3 prefers to compete against $\min[v_1, v_2]$ than against $\max[v_1, v_2]$, so he bids the valuation in auction 2 in round 2. Once, he does so, the other buyer prefers to stick to bidding $b(v_i)$, as otherwise they would risk paying $v^{2:3}$, which is too much. Because they risk their bids being rejected in auction 2, the potential gain of winning the good in auction 2 decreases. Therefore, they are willing to bid more aggressively in auction 1 as compared to equilibrium presented in proposition 4.2. As a result, auction 1 finishes with the expected price $E[p_1] > E[v^{3:3}]$ and auction 2 finishes with the expected price $E[p_2] < E[v^{3:3}]$. The following proposition presents a corresponding equilibrium.

Proposition 4.3. *Define $\tilde{\sigma}_i$ as follows:*

(1) *at $t = 1$ do not bid,*

(2) *at* $t = 2$ *bid* $b(v_i) = v_i - \alpha \int_0^{v_i} F(v_j)dv_j$ *in auction* 1*,*

(3) *at* $t = 3$ *bid* v_i

  (a) *in auction* 1*, if you have not lost in auction* 1 *and have not won in auction* 2*,*

  (b) *in auction* 2*, otherwise,*

and $\bar{\sigma}_3$ *as follows:*

(1) *at* $t = 2$ *bid* v_3 *in auction* 2*,*

(2) *at* $t \neq 2$ *do not bid,*

Then, $(\tilde{\sigma}_1, \tilde{\sigma}_2, \bar{\sigma}_3)$ *is a Bayesian Nash equilibrium of* Γ^1*. The equilibrium outcome is inefficient. The expected price in auction 1 (*$E[b(v^{2:2})]$*) is higher than the expected price in auction 2 (*$\alpha E[v^{3:3}]$*).*

Proof. See appendix. □

To conclude, this subsection presents an inefficient equilibrium with last-minute bidding. In this equilibrium, buyers start the coordination late and might end up in an inefficient allocation. By changing identities of auctions or bidders, it is easy to construct other equilibria. The next subsection discusses all the possible equilibrium outcomes.

4.2.4. Summary of possible equilibrium outcomes

The previous subsections concentrated on the strategic interactions in the dynamic framework. All the presented equilibria can be easily changed to define another equilibria. For example, the identity of bidders and auctions could be changed. Some irrelevant bids could be added. The important thing is that under some conditions the changes do not lead to different equilibrium outcomes. These conditions are: (1) bidders bid according to strictly increasing bidding functions or do not bid at all, (2) the current winner in one auction does not bid in the other auctions and (3) at least two bidders follow the same strategy. In other words, I concentrate on relatively simple quite symmetric monotonic equilibria. The following proposition gives an overview of the possible equilibrium outcomes.

Proposition 4.4. *Let* $(\sigma_1, \sigma_2, \sigma_3)$ *be a pure Bayesian Nash equilibrium (of* Γ^1*) in undominated strategies in which:*

(1) $\frac{\partial}{\partial v_i} b^i_{at}(v_i) > 0$ *or* $b^i_{at}(v_i) = 0$ *for all* i, t *and* a,

(2) *no current winner in one auction bids in the opposite auction,*

(3) *at least two bidders follow the same strategy.*

Define the following outcomes:

(1) *(outcome 1) efficient outcome with the expected price of* $E[v^{3:3}]$ *in both auctions,*

(2) *(outcome 2) the highest value player always wins an object; the second highest value bidder wins an object with probability* $\frac{2}{3}\alpha$ *and the lowest value bidder wins an object with probability* $\frac{1}{3}(1-\alpha)$*, expected final prices are given by* $E[v^{2:2} - \alpha \int_0^{v^{2:2}} F(v_j)dv_j]$ *and* $E[v^{3:3}]$.

Then, $(\sigma_1, \sigma_2, \sigma_3)$ *leads to outcome 1 or outcome 2.*

Proof. See Appendix. □

Having more periods would give more flexibility in the timing of the crucial bids and add some space for irrelevant bids, but does not affect the final outcome. In equilibrium, some bidders would still test the ordering of their valuations and later appropriately reallocate. There would be efficient equilibria without last-minute bidding and inefficient equilibria with last-minute bidding. In an efficient equilibrium, the expected price would equal to the lowest valuation. In an inefficient equilibrium, the expected prices would be given by $E[v^{2:2} - \alpha \int_0^{v^{2:2}} F(v_j)dv_j]$ and $E[v^{3:3}]$.

The logic behind presented equilibria can be easily applied to identify equilibria of the model with a larger number of auctions and bidders. Suppose there are N bidders and A auctions, satisfying $\infty > N > A \geq 2$. Then, it is possible to construct the following equilibrium. First, every bidder bids $b^i_{11}(v_i)$ in auction 1. Second, the winning bidder bids v_i in auction 1 and losing bidders bid $b^i_{22}(v_i)$ in auction 2. Third, the winning bidder bids v_i in auction 2 and losing bidders bid $b^i_{33}(v_i)$ in auction 3. The procedure repeats until all auctions are filled. The bidding functions are derived by solving $Ev^{M+1:N} = Eb^i_{AA}(v^{M+1:N}) = Eb^i_{A-1A-1}(v^{M:N}) = ... = Eb^i_{11}(v^{2:N})$. Bids of winners are important, as they assure that no losing bidder has incentives to bid again in the same auction, which allows to keep prices at reasonable level.

It is also possible to have an efficient equilibrium in which $N-1$ (where $N > 3$) bidders test ordering of their valuations in auction 1 and one bidder (say buyer 1) bids the valuation in auction 2. Since bidder 1 bids the valuation in auction, no-one is willing to risk winning the good at the price of $v^{2:N}$ by bidding in auction 2. Each buyer $i \neq 1$ has incentives to bid $b(v_i) < v_i$ in auction 1. If he loses, he learns that his valuation is not higher than $v^{2:N}$, so that he has no chance to win against the winning bidder. Bidding in auction 2 is then his last chance to win the good. He weakly prefers to bid his valuation there. By comparing the expected gain from winning the good in auction 1 and in auction 2, one derives optimal $b(v_i)$.

It is more difficult to identify an equilibrium where $N-x$ (where $N > 3$ and $1 < x < N-1$) bidders test ordering of their valuations in auction 1 and the remaining x buyers bid in auction 2. Since $x > 1$, bidders do not wish to bid their valuations in auction 2 before learning some information on the ordering of the valuations. Hence, in both auction bidders must test the ordering of their valuations. The losing bidders from auction 1 want to bid their valuations in auction 2. Similarly, the losing bidders from auction 2 want to bid their valuations in auction 1. The problem becomes complicated and the optimal bidding function might be difficult to identify.

Finding an asymmetric equilibrium with sniping is even more complicated. Beside all the possible reallocations, bidders need to take care of the possible rejections of some of their bids. Hence, checking for all the possible deviations might be quite demanding process.

To conclude, this section shows that bidders wish to coordinate between the auctions so that no-one will pay too much. They do so by testing the ordering of their valuations.

If they do it early enough, they arrive at an efficient outcome. Otherwise, they have to send late risky bids and the outcome might be inefficient. This subsection also provides a discussion how the model extends with more bidders, auctions or rounds.

4.3. Overlapping Internet auctions

The previous section studied simultaneous Internet auctions. In this model, the presence of multiple rounds gives a lot of flexibility to the bidders. This flexibility gives arise to multiple equilibria. All these equilibria have two components (1) test of the ordering of the valuation and (2) reallocation of the losing bidders. When (1) happens at stage 1, (2) follows at stage 2. All the bids are accepted and the resulting outcome is efficient. When (1) takes place at stage 2, (3) is scheduled at stage 3. At stage 3, some of the bids are rejected and the resulting outcome might be inefficient. Inefficiency is closely related with the simultaneous schedule of the auctions. This section argues that if auctions are overlapping, the reallocation always happens under the safe bid transmission.

4.3.1. The model

Consider overlapping Internet auctions. The model is modified in the following way. All the rules remain the same, but the schedule of the auctions changes. Let t $(t = 1, ..4)$ denote a period. Auction 1 lasts from $t = 1$ to $t = 3$ and auction 2 takes place from $t = 2$ to $t = 4$. Each bid exceeding the current standing price is accepted at $t = 1$ and $t = 2$ in auction 1 and at $t = 2$ and $t = 3$ in auction 2. Each bid exceeding the current standing price is accepted with probability $\alpha \in (0, 1)$ at $t = 3$ in auction 1 and at $t = 4$ in auction 2. The rules defining possible bids, winners, prices and payoffs remain unchanged. The new game is denoted by Γ^2.

4.3.2. Results

The trade-offs in the revised model are very similar to the trade-offs in the original model. Bidders want to learn a little about the ordering of their valuations to appropriately coordinate between the auctions. The motives behind efficient equilibria are the same. Take the equilibrium presented in proposition 4.1. In this equilibrium buyers first bid according to the bidding function given by (1) in auction 1. Afterwards, the winning bidder bids the valuation in auction 1 and the losing bidders bid their valuations in auction 2. All bids arrive under safe bid transmission. Clearly, this is still an equilibrium of the modified model, which the next proposition shows.

Proposition 4.5. *Define $\tilde{\sigma}_i$ as in proposition 4.1. Then, $(\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3)$ is a Bayesian Nash equilibrium of Γ^2. The equilibrium outcome is efficient. The expected price in auction 1 and the expected price in auction 2 equal to $E[v^{3:3}]$.*

Proof. The proof easily follows from the proof of proposition 4.1. □

Take now the equilibrium presented in proposition 4.2. In this equilibrium, two bidders test the ordering of their valuations in auction 1 and later appropriately reallocate. Meanwhile, the third buyer bids the valuation in auction 2. All bids arrive under safe bid transmission. Clearly, this is still an equilibrium of the modified model, which the next proposition shows.

Proposition 4.6. *Define $\tilde{\sigma}_i$ and $\bar{\sigma}_3$ as in proposition 4.2. Then, $(\tilde{\sigma}_1, \tilde{\sigma}_2, \bar{\sigma}_3)$ is a Bayesian Nash equilibrium of Γ^2. The equilibrium outcome is efficient. The expected price in auction 1 and the expected price in auction 2 equal to $E[v^{3:3}]$.*

Proof. The proof easily follows from the proof of proposition 4.2. □

Efficient equilibria of Γ^1 remain equilibria of Γ^2. Now, the question is what happens with inefficient equilibria.

Take an equilibrium presented in proposition 4.3. In this equilibrium, two players test the ordering of their valuations before the last stage and allocate in the risky period. I check whether Γ^2 has a similar equilibrium. There are two possibilities. First, the test of the ordering of the valuations could be made at $t = 3$ and reallocation could take place at $t = 4$. This is not optimal, as some player would have incentives to submit the first bid at $t = 2$. Second, the test could be made at $t = 2$ and the reallocation could take place at $t = 4$. Again, this is not optimal. Some buyer would have incentives to reallocate at $t = 3$ instead of $t = 4$.

All in all, in the presence of overlapping auctions only efficient equilibria persist. The next proposition gives the result.

Proposition 4.7. *Let $(\sigma_1, \sigma_2, \sigma_3)$ be a pure Bayesian Nash equilibrium (of Γ^2) in undominated strategies in which:*

(1) $\frac{\partial}{\partial v_i} b^i_{at}(v_i) > 0$ *or* $b^i_{at}(v_i) = 0$ *for all* i*,* t *and* a*,*

(2) *no current winner in one auction bids in the opposite auction,*

(3) *at least two bidders follow the same strategy.*

Then, $(\sigma_1, \sigma_2, \sigma_3)$ leads to the efficient outcome with expected price of $E[v^{3:3}]$ in both auctions.

Proof. See Appendix. □

In contrast to simultaneous auctions, overlapping auctions have no inefficient equilibria. The expected revenue from the two auctions is given by $2E[v^{3:3}]$. Simultaneous auctions also have efficient equilibria rising the revenue of $2E[v^{3:3}]$. What's more, they have inefficient equilibria with the revenue of at least of $(1+\alpha)E[v^{3:3}]$. One auction yields the price above $E[v^{3:3}]$ and the other auction has the expected price of $\alpha E[v^{3:3}]$. Hence, when auctions are simultaneous, auction 1 benefits and auction 2 suffers a loss. The total revenues are more difficult to compare. For the uniform distribution, last-minute bidding harms the total revenue.

To conclude, in competing auctions, buyers wish to coordinate. When they start the coordination in simultaneous auctions, they might end up bidding late, so that some of their bids are rejected. The resulting outcome is inefficient. The revenue might be harmed. The seller gains from simultaneous auction, when bidders test the ordering of their valuations in his auction. In overlapping auctions, there is always time to safely reallocate. The resulting outcome is efficient and the two auctions have equal expected prices. Hence, overlapping auctions are better for efficiency. The revenue from a given auction is ambiguous.

4.4. Conclusions

The present paper demonstrates a dynamic model of independent private value simultaneous or overlapping Internet auctions. It shows how multiple and last-minute bidding lead to the equilibrium outcomes.

One of the main achievement of this paper is to rationalize multiple bidding in Internet auctions. Most of the existing studies show that sniping is a unique equilibrium strategy. In some studies, in the presence of a fixed end time a bidder is indifferent between sending one and several bids. In my model, multiple bidding is part of a strategic interaction and is essential for sustaining some of the equilibrium outcomes. I show that in an equilibrium players first bid less aggressively and afterwards the winning bidder bids the valuation in the same auction and the losing bidder bids the valuation in the opposite auction.

I also compare simultaneous and overlapping Internet auctions. I observe that all the inefficiencies observed in the simultaneous auctions disappear in overlapping auctions. Therefore, the planner interested in efficiency should opt for overlapping auctions. The seller who cares about the revenue should take into account that simultaneous auctions can produce an equilibrium in which the revenue of one auction increases and the revenue of the other auction decreases. This equilibrium is not present in overlapping auctions. The seller who faces competition from the other sellers might lose when setting the ending time equal to the competing auction, as his auction might be perceived as the final destination of the bidders. He might be also lucky to increase his revenue, when bidders notice his auction as the first one.

4.5. Appendix

Proof. (proposition 4.1) Suppose that each player $j \neq i$ follows $\hat{\sigma}_j$. Assume for a moment that player i becomes the current winner in auction 1 at $t = 2$. Then, other players will not bid in auction 1 any more and he will remain a current winner there. Hence, he weakly prefers to follow $\hat{\sigma}_i$ till the end of the game.

If player i does not become a current winner at auction 1 at $t = 2$, he knows that a bid of $\max_{j\neq i} v_j$ will be submitted in auction 1 and a bid of $\min_{j\neq i} v_j$ will be submitted in auction 2. Clearly, he is weakly better off, when he bids his valuation in auction 2 before the last stage. Once he does so, his expected payoff cannot be further increased. Hence, he also follows $\hat{\sigma}_i$ till the end of the game.

Concluding, when each player $j \neq i$ follows $\hat{\sigma}_j$ during the whole game, then player i cannot do better than to follow $\hat{\sigma}_i$ as from $t = 2$.

Now, suppose that as from $t = 2$ player i follows $\hat{\sigma}_i$ and that each player $j \neq i$ follows $\hat{\sigma}_j$ during the whole game. At $t = 1$, player i submits the bid $\varphi(v_i) = c$ that maximizes his expected payoff:

$$2\int_0^{b^{-1}(c)}\int_0^{v_2}(v_i - b(v_2))dF(v_3)dF(v_2) + 2\int_{b^{-1}(c)}^{1}\int_0^{v_i}(v_i - v_3)dF(v_3)dF(v_2) - I$$

where:

$$I = \begin{cases} \int_{b^{-1}(c)}^{v_i}\int_{b^{-1}(c)}^{v_i}(v_i - v_3)dF(v_3)dF(v_2) & \text{if } b^{-1}(c) < v_i \\ 0 & \text{otherwise} \end{cases}$$

The first order condition implies:

$$\frac{db^{-1}(c)}{dc}\cdot\left(2f(b^{-1}(c))\left(\int_0^{b^{-1}(c)}(v_i - c)dF(v_3) - \int_0^{v_i}(v_i - v_3)dF(v_3)\right) - \frac{dI}{dc}\right) = 0$$

By symmetry $c = b(v_i)$ and as a consequence, $b^{-1}(c) = v_i$. Hence, we obtain:

$$b(v_i) = \frac{1}{F(v_i)}\int_0^{v_i} v_3 f(v_3)dv_3 \tag{4.3}$$

Using integration by parts, we rewrite (4.3) as:

$$b(v_i) = v_i - \frac{\int_0^{v_i} F(v_3)dv_3}{F(v_i)}$$

Now, we prove that player i prefers to bid $b(v_i)$ instead of any b_i. Clearly, it is never optimal for player i to $b_i > b(1)$. As b is strictly increasing, and continuous, a bid $0 \leq b_i \leq b(v)$ corresponds to a unique value x for which $b(x) = b_i$. Suppose that $x > v_i$. Then, we can write bidder i's expected profit from bidding b_i as:

$$EU_i(x, v_i) = EU_i(v_i, v_i) + \Omega$$

where:

$$\Omega = 2\int_{v_i}^{x}\left(\int_0^{v_2}(v_i - b(v_2))dF(v_3) - \int_0^{v_i}(v_i - v_3)dF(v_3)\right)dF(v_2)$$

By using $b(v_2) = \frac{1}{F(v_2)}\int_0^{v_2} v_1 f(v_1)dv_1$ and rearranging, we obtain:

$$\Omega = 2\int_{v_i}^{x}\left(\int_{v_i}^{v_2} v_i dF(v_3) - \int_0^{v_2}\int_0^{v_2}\frac{v_1}{F(v_2)}dF(v_1)dF(v_3) + \int_0^{v_i} v_3 dF(v_3)\right)dF(v_2)$$

Further rearranging gives:

$$\Omega = 2\int_{v_i}^{x}\int_{v_i}^{v_2}(v_i - v_1)dF(v_1)dF(v_2)$$

Since $x > v_i$:

$$\Omega < 0$$

Hence, bidder i cannot profitably deviate by bidding more than $b(v_i)$.

Suppose now that $x < v_i$. Then, we can write bidder i's expected profit from bidding b_i as:

$$EU_i(x, v_i) = EU_i(v_i, v_i) + 2\int_x^{v_i}\Psi dF(v_2)$$

where:

$$\begin{aligned}\Psi &= \int_0^{v_2}\left(\int_0^{v_2}\frac{v_1}{F(v_2)}dF(v_1) - v_i\right)dF(v_3) + \int_0^{v_i}(v_i - v_3)dF(v_3) - \int_x^{v_i}\frac{v_i - v_3}{2}dF(v_3)\\ &= \int_0^{v_2}\int_0^{v_2}\frac{v_1 - v_i}{F(v_2)}dF(v_1)dF(v_3) + \int_0^{v_i}\int_0^{v_2}\frac{v_i - v_3}{F(v_2)}dF(v_1)dF(v_3) - \int_x^{v_i}\frac{v_i - v_3}{2}dF(v_3)\end{aligned}$$

By rearranging, one finds that:

$$\begin{aligned}\Psi &< \int_0^{v_2}\int_0^{v_2} \frac{v_1 - v_3}{F(v_2)} dF(v_1)dF(v_3) - \int_x^{v_i} \frac{v_i - v_3}{2} dF(v_3) \\ &= \int_x^{v_i} \frac{v_i - v_3}{2} dF(v_3) \\ &< 0\end{aligned}$$

Hence, bidder i cannot profitably deviate by bidding less than $b(v_i)$.

Concluding, I have shown that at every t a player i cannot profitably deviate from following $\hat{\sigma}_i$, if he knows that everybody else plays $\hat{\sigma}_i$. Hence, $(\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3)$ is a Bayesian Nash equilibrium.

Suppose now that everyone plays according to $(\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3)$. Then, clearly the resulting outcome is efficient. The expected price at auction 2 is given by:

$$E[p_2] = E[v^{3:3}]$$

The expected price at auction 1 equals:

$$\begin{aligned}\int_0^1 b(v_2)dF^{2:3}(v_2) &= 6\int_0^1\int_0^{v_i} v_3 f(v_3)dv_3(1 - F(v_2))f(v_2)dv_2 \\ &= 3\int_0^1 v_3(1 - F(v_3))^2 f(v_3)dv_3 \\ &= E[v^{3:3}]\end{aligned}$$

where the first equality is found by using (4.3) and the second equality is found by exchanging the order of integrals. □

Proof. (proposition 4.2) The optimally of the behavior of player 3 can be proven by repeating the argument of the proposition 4.3 and plugging $\alpha = 1$, where necessary.

Now, I prove that it is optimal for player 1 to follow $\tilde{\sigma}_1$, if other players follow strategies defined in the proposition. If he wins in a given auction, then it is optimal for him not to bid in the other auction, as he is already sure about winning the object. If he has not bid before $t = 2$, it is then his weakly dominant strategy to bid v_1 in auction 1 at $t = 2$. If he has bid $\vartheta(v_1) = \beta$ in auction 1 at $t = 1$ and has lost, he prefers to bid v_i in auction 2 at $t = 2$, as his expected payoff from bidding in auction 1 ($\int_{\min[b^{-1}(\beta),v_i]}^{v_1}(v_1 - v_2)dF(v_2),$) is lower than his expected payoff from bidding in auction 2 ($\int_0^{v_1}(v_1 - v_3)dF(v_3)$). All in all, I have shown that no matter what happens at $t = 1$, it is optimal for player 1 to follow $\tilde{\sigma}_1$ at $t > 1$.

The optimality of $b(v_1)$ can be proven by repeating the argument of lemma 4.2, plugging $\alpha = 1$, where necessary and appropriately changing the timing.

Concluding, I have shown that if player 2 and player 3 play according to $\tilde{\sigma}_2$ and $\bar{\sigma}_3$ respectively, it is optimal for player 1 to follow $\tilde{\sigma}_1$. Finally, note that due to symmetry player 2 follows $\tilde{\sigma}_2$, if player 1 and player 3 follow $\tilde{\sigma}_1$ and $\bar{\sigma}_3$ respectively.

The expected price at auction 2 is given by:

$$Ep_2 = Ev^{3:3}$$

The expected price at auction 1 is given by:

$$\begin{aligned}
Ep_1 &= Eb(v^{2:2}) \\
&= \int_0^1 \left(v_1 - \int_0^{v_1} (v_1 - v_3) f(v_3) dv_3 \right) f^{2:2}(v_1) dv_1 \\
&= \int_0^1 v_1(1 - F(v_1)) f^{2:2}(v_1) dv_1 + \int_0^1 v_3(1 - F^{2:2}(v_3)) f(v_3) dv_3 \\
&= 2 \int_0^1 v_1(1 - F(v_1))^2 f(v_1) dv_1 + \int_0^1 v_3(1 - F(v_3))^2 f(v_3) dv_3 \\
&= 3 \int_0^1 v_1(1 - F(v_1))^2 f(v_1) dv_1 \\
&= E[v^{3:3}] \\
&= Ep_2
\end{aligned}$$

The equilibrium outcome is efficient, as the player with $v_i = \max[v_1, v_2]$ wins the object at auction 1 and the player with $\max[\min[v_1, v_2], v_3]$ wins the object at auction 2. □

Proof. (proposition 4.3) Suppose that player 1 and player 2 play according to $\tilde{\sigma}_1$ and $\tilde{\sigma}_2$ respectively. Then, player 3 prefers to bid in auction 2, as in auction 1 he would have to compete against $(1-\alpha)f(\max[v_1, v_2]) + \alpha \max[v_1, v_2]$, which is more than the highest bid submitted by his opponents in auction 2 ($(1-\alpha)0 + \alpha \min[v_1, v_2]$). Once he decides for bidding in auction 2, bidding v_3 at $t = 2$ weakly dominates all the other possible bids in auction 2. All in all, it is optimal for player 3 to follow $\bar{\sigma}_3$.

Now suppose that player 2 and player 3 play according to $\tilde{\sigma}_2$ and $\bar{\sigma}_3$ respectively. Note that if player 1 wins in a given auction, then it is optimal for him not to bid in the other auction, as he is already sure about winning the object. Hence, he cannot do better than to follow $\tilde{\sigma}_1$.

Now suppose that player 1 has not bid before $t = 3$. Then, he prefers to bid first in auction 1, as the expected value of the highest bid in auction 2 (given by $E(v_3)$) is higher than the expected value of the highest bid in auction 1 $\left(\text{given by } E\left(v_2 - (1-\alpha)\alpha \int_0^{v_2} F(v_j)dv_j\right)\right)$. It is then his weakly dominant strategy to bid v_1 in auction 1. In other words, he cannot do better than to follow $\tilde{\sigma}_1$.

Now suppose that player 1 has lost in auction 1 before $t = 3$. Since he has lost, he infers that $v_1 < v_2$. Furthermore, he has a weakly dominant strategy to bid v_1 in the optimally chosen auction. Suppose he bids in auction 1. Then, he will win the good in auction 1 only if his bid is accepted, $v_1 > b(v_2)$ and the bid of player 2 is rejected in auction 1. Therefore, his expected payoff may be written as:

$$\pi_1^* = \alpha(1-\alpha) \int_{v_1}^{b^{-1}(v_1)} \left(v_1 - v_2 + \alpha \int_0^{v_2} F(v_3)dv_3\right) dF(v_2)$$

By rearranging, one obtains:

$$\pi_1^* < \alpha^2(1-\alpha) \int_{v_1}^{b^{-1}(v_1)} \int_0^{v_2} F(v_3)dv_3 dF(v_2)$$

By changing the order of integrals, one finds that the above expression is equal to:

$$\alpha^2(1-\alpha)\int_0^{v_1}\int_{v_3}^{b^{-1}(v_1)} dF(v_2)F(v_3)dv_3$$

Further manipulation yields:

$$\begin{aligned}
\pi_1^* &< \alpha^2(1-\alpha)\int_0^{v_1}(F(b^{-1}(v_1)) - F(v_3))F(v_3)dv_3 \\
&< \alpha^2(1-\alpha)\int_0^{v_1} F(v_3)dv_3 \\
&< \alpha\int_0^{v_1} F(v_3)dv_3 \\
&= \alpha\int_0^{v_1}(v_1 - v_3)f(v_3)dv_3 \\
&= \pi_2^*
\end{aligned}$$

where π_2^* is the payoff player 1 obtains when he bids the valuation in auction 2. π_2^* is shown to be smaller than π_1^*, which means that player 1 prefers to move to auction 2 after losing in auction 1. In other words, he cannot do better than follow $\tilde{\sigma}_1$.

All in all, I have shown that no matter what happens before $t = 3$, it is optimal for player 1 to follow $\tilde{\sigma}_1$ at $t = 3$.

Suppose now that everyone follows the strategy specified in the proposition, but player 1 bids $\vartheta(v_1) = \beta$ instead of $b(v_1)$. Then, his expected payoff is given by:

$$\int_0^{b^{-1}(\beta)}(v_1 - b(v_2))f(v_2)dv_2 + \alpha\int_{b^{-1}(\beta)}^1\int_0^{v_1}(v_1 - v_3)f(v_3)dv_3f(v_2)dv_2$$

The first order condition implies:

$$\frac{db^{-1}(\beta)}{d\beta}\left((v_1 - b(b^{-1}(\beta)))f(b^{-1}(\beta)) - \alpha\int_0^{v_1}(v_1 - v_3)f(v_3)dv_3f(b^{-1}(\beta))\right) = 0$$

By symmetry:

$$b(v_1) = v_1 - \alpha\int_0^{v_1}(v_1 - v_3)f(v_3)dv_3$$

Integrating by parts, I find:

$$b(v_1) = v_1 - \alpha \int_0^{v_1} F(v_3) dv_3$$

Hence, $b(v_i)$ is a natural candidate for the equilibrium bidding function in auction 1 at $t = 2$. I now show that player 1 cannot do better than to bid $b(v_1)$ in auction 1 at $t = 2$. Imagine he bids $b(x)$ instead of $b(v_1)$. Then, his expected payoff equals to:

$$U_1(v_1, x) = \int_0^x (v_1 - b(v_2)) f(v_2) dv_2 - \alpha \int_x^1 \int_0^{v_1} (v_1 - v_3) f(v_3) dv_3 f(v_2) dv_2$$

I differentiate $U_1(v_1, x)$ with respect to x:

$$\begin{aligned} \left. \frac{\partial U_1(v_1, x)}{\partial x} \right|_{x=v_1} &= f(x) \left(v_1 - b(x) - \alpha \int_0^{v_1} (v_1 - v_3) f(v_3) dv_3 \right) \\ &= 0 \end{aligned}$$

I check the second order condition:

$$\begin{aligned} \left. \frac{\partial^2 U_1(v_1, x)}{\partial x \partial x} \right|_{x=v_1} &= f'(x) \left(v_1 - b(x) - \alpha \int_0^{v_1} (v_1 - v_3) f(v_3) dv_3 \right) - f(x) b'(x) \\ &= -f(x)^2 (1 - \alpha F(x)) \\ &< 0 \end{aligned}$$

Hence, player 1 cannot do better than to set $x = v_1$.

Concluding, I have shown that if player 2 and player 3 play according to $\tilde{\sigma}_2$ and $\bar{\sigma}_3$ respectively, it is optimal for player 1 to follow $\tilde{\sigma}_1$.Finally, note that due to symmetry player 2 follows $\tilde{\sigma}_2$, if player 1 and player 3 follow $\tilde{\sigma}_1$ and $\bar{\sigma}_3$ respectively.

The expected price at auction 2 is given by:

$$Ep_2 = \alpha E v^{3:3}$$

The expected price at auction 1 is given by:

$$
\begin{aligned}
Ep_1 &= Eb(v^{2:2}) \\
&= \int_0^1 \left(v_1 - \alpha \int_0^{v_1} (v_1 - v_3) f(v_3) dv_3 \right) f^{2:2}(v_1) dv_1 \\
&= \int_0^1 v_1 (1 - \alpha F(v_1)) f^{2:2}(v_1) dv_1 + \alpha \int_0^1 v_3 (1 - F^{2:2}(v_3)) f(v_3) dv_3 \\
&> 2\alpha \int_0^1 v_1 (1 - F(v_1))^2 f(v_1) dv_1 + \alpha \int_0^1 v_3 (1 - F(v_3))^2 f(v_3) dv_3 \\
&= 3\alpha \int_0^1 v_1 (1 - F(v_1))^2 f(v_1) dv_1 \\
&= \alpha E[v^{3:3}] \\
&= Ep_2
\end{aligned}
$$

Since the bid of the player bidding in auction 2 at $t = 3$ the equilibrium outcome might be inefficient. □

The following two lemmas are needed for the proof of proposition 4.4.

Lemma 4.8. *Let $\alpha < 1$. Define $\tilde{\sigma}_i$ by:*

(1) *at $t < 2$ do not bid*

(2) *at $t = 2$ bid $b(v_i)$, where $b'(v_i) > 0$, in auction 1,*

(3) *at $t = 3$ bid v_i at auction 2, if you are a current winner in auction 1 and in auction 2, otherwise,*

Then, $(\tilde{\sigma}_1, \tilde{\sigma}_2, \tilde{\sigma}_3)$ is not a Bayesian Nash equilibrium.

Proof. Suppose that $(\tilde{\sigma}_1, \tilde{\sigma}_2, \tilde{\sigma}_3)$ is a Bayesian Nash equilibrium. Then, the expected payoff of player 1 who deviates by bidding $b(y)$ instead of $b(v_1)$, given by $R = 2\Gamma + 2\Delta + I$, where:

$$\Gamma \equiv \int_0^{b^{-1}(\beta)} \int_0^{v_2} (v_1 - b(v_2)) dF(v_3) dF(v_2)$$

$$\Delta \equiv \alpha^2 \int_{b^{-1}(\beta)}^{1} \int_0^{v_1} (v_1 - v_3) dF(v_3) dF(v_2) + \alpha(1-\alpha) \int_{b^{-1}(v_1)}^{1} \int_0^{v_2} v_1 dF(v_3) dF(v_2)$$

$$I \equiv \begin{cases} \alpha^2 \int_{b^{-1}(\beta)}^{v_i} \int_{b^{-1}(\beta)}^{v_i} (v_i - v_3) dF(v_3) dF(v_2) & \text{if } b^{-1}(\beta) < v_i \\ 0 & \text{otherwise} \end{cases}$$

The first order condition implies:

$$\int_0^y (v_1 - b(y) dF(v_3) - \alpha^2 \int_0^{v_1} (v_1 - v_3) dF(v_3) - \alpha(1-\alpha) \int_0^{v_2} v_1 dF(v_3) + \frac{dI}{dy} = 0$$

Substituting $y = v_i$, I find:

$$\int_0^{v_1} ((1-\alpha)v_1 - b(v_1) dF(v_3) + \alpha^2 \int_0^{v_1} v_3 dF(v_3) = 0$$

or equivalently:

$$b(v_1) = (1-\alpha)v_1 + \frac{\alpha^2}{F(v_1)} \int_0^{v_1} v_3 dF(v_3)$$

which using integration by parts can be written as:

$$b(v_1) = (1 - \alpha(1-\alpha))v_1 - \frac{\alpha^2 \int_0^{v_1} F(v_3) dv_3}{F(v_1)} \tag{4.4}$$

Now, I check whether it is profitable for player 1 to deviate by bidding v_1 at auction 2 at $T-1$. Note that if $v_1 \neq v^{1:3}$, player 1 would increase his expected payoff from $\alpha E[(\Pr[v_1 > \min[v_2, v_3](v_2 - \alpha v_3) + \Pr[v_1 < \min[v_2, v_3](1-\alpha)v_2)] \equiv R^D$ to $\frac{R^D}{\alpha}$. Therefore, in order to arrive at a contradiction, it is enough to show that the proposed deviation is profitable, when $v_1 = v^{1:3}$. Note the following:

$$\begin{aligned} E[p^1 | v_1 &> v^{1:2}] = E\left[b(v^{1:2}) | v_1 > v^{1:2}\right] \\ &= \int_0^{v_1} \left((1-\alpha(1-\alpha))v_2 - \frac{\alpha^2 \int_0^{v_2} F(v_3) dv_3}{F(v_2)} \right) dF^{1:2}(v_2) \\ &> \alpha \int_0^{v_1} \left(v_2 - \frac{\int_0^{v_2} F(v_3) dv_3}{F(v_2)} \right) dF^{1:2}(v_2) \\ &= \alpha E[v^{2:2} | v_1 > v^{1:2}] \\ &= E[p^2 | v_1 > v^{1:2}] \end{aligned}$$

A contradiction. □

Lemma 4.9. *A strategy profile $(\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3)$ in which each player waits with bidding until the last stage and never bids above the valuation is not a pure perfect Bayesian Nash equilibrium.*

Proof. Suppose that $(\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3)$ is a Bayesian Nash equilibrium. Then, $\hat{\sigma}_i$ must imply that player i bids v_i at $t = 3$. This is because bidding less only decreases a chance for a profitable transaction and bidding more is not allowed. Furthermore, it cannot happen that all three bidders bid at the same auction, as one of them would be better off, if he deviated and bid his valuation at the opposite auction at the last stage. Hence, two players have to bid their valuations together in one auction and the third one has to bid his valuation in the opposite auction. Take a player who is not the only bidder in auction a. Without loss of generality assume that it is player 3 and that he bids against player 1 at auction 1. Suppose that he deviates and bids in auction 1 before T. Note that other players will not know the value of his bid then. Therefore, after seeing the deviation, they can base their decision about punishing him only on the pure fact that his bid was accepted. They could try to punish him by bidding their valuations in auction 1. This would not heart player 3, as he could set $\hat{b}^3_{12}$ very close to 0 and bid his valuation at auction 2 at T. Hence, I need to look at other possible reactions to the opponent's deviation. There are three other possibilities: nobody bids at auction 1 again; player 1 bids v_1 at auction 1 at and player 2 bids v_2 at auction 2 and finally player 1 bids v_1 at auction 2 at and player 2 bids v_2 at auction 1. In the first two cases player 3 is already better off, when he bids his valuation at auction 1 at $t = 2$ instead of waiting with bidding for the last stage. Then, his bid is accepted with certainty and the expected price he pays in case of winning does not increase. In case 1 it is even lower, as it equals to zero. In case 3 his deviation does not decrease his expected as $E[v_2] = E[v_1]$. Hence, a contradiction. □

Proof. (proposition 4.4) Let $(\sigma_1, \sigma_2, \sigma_3)$ satisfy the required properties. I start from the last stage and ask if it could happen that no information was revealed before the last stage. Since at least two players follow the same strategy and $\frac{\partial}{\partial v_i} b^i_{at}(v_i) > 0$ or $b^i_{at}(v_i) = 0$ for all i, t and a, it must be the case that two bidders did not bid before the last stage. Lemma 4.9 shows that it cannot be that all the bidders did not bid before the last stage. Suppose then that buyer 1 bid b_1 in auction 1 before the last stage and buyers 2 and 3 did not bid before the last stage. Since last stage are their last chance to win the good, they will bid their valuations in the last stage. Then, player 1 prefers to bid his valuation in auction 1 before the last stage. If so, buyers 2 and 3 prefer to bid their valuations in auction 2. But then, buyer 2 could profitably deviate by bidding the valuation in auction 2 in stage 2. Hence, this cannot be an equilibrium. Buyers need to have some information on the ordering of the valuations before the last stage.

Without loss of generality suppose that before $t = 3$ it has been revealed that player 1 has higher valuation than player 2. Since $\frac{\partial}{\partial v_i} b^i_{at}(v_i) > 0$ or $b^i_{at}(v_i) = 0$ for all i, t and a, it must be the case that player 1 and player 2 have bid in the same auction and player 2 has been overbid. Suppose no-one else has bid there. Then, it can happen that a current winner in auction 1 bids the valuation in auction 1 and the other bidders bid the valuations in auction 2, similarly as in proposition 4.2. The resulting outcome is efficient and the bidder with the lowest possible valuation always obtains payoff of zero. Hence, the revenue equivalence theorem implies that expected prices are equal to $E[v^{3:N}]$ in both auctions. In other words, outcome 1 arises. It remains to explain what happens if player 1 bids in auction 2 at $t = 3$. This can happens only when at $t = 2$ he is overbid in auction 1 by player 3. Then, he weakly prefers to bid the valuation in auction 2 at $t = 3$. When he does so, player 2 weakly prefers to bid v_2 in auction 1 by $t = 3$. But this makes player 3 unwilling to outbid player 1 in auction 1 and hence, no such equilibrium is sustained.

If the ordering of the two valuations is revealed in auction 1 in stage 2, then the current winner cannot bid in the auction 2. Hence, he prefers to bid the valuation in auction 1. If so, the losing bidder prefers to bid the valuation in the opposite auction.

The third bidder prefers to fight against the losing bidder, so he also bids the valuation in auction 2. As a result, the trade-offs are the same as in proposition 4.2. Hence, outcome 2 must arise.

Suppose now that the ordering of all valuations has been revealed before $t = 3$. Lemma 4.8 shows that it cannot happen that the ordering was revealed at $t = 2$. Hence, bidders must know the ordering at $t = 1$. Then, bidders who do not have the highest valuation always bid in the other auction than the bidder with the highest valuation. Furthermore, they weakly prefer to bid their valuations there. As a result, the resulting outcome is efficient and hence, by the revenue equivalence theorem expected price equals to $Ev^{3:N}$ in both auctions. In other words, outcome 1 arises. □

Proof. (proposition 4.7) Let $(\sigma_1, \sigma_2, \sigma_3)$ satisfy the required properties. First I prove that there does not exist an equilibrium in which bidders bid under risky bid transmission. Note that a buyer who bids only in auction 2 has no reason to bid at $t = 4$. He will always bid under certain bid transmission. Additionally, he might send some irrelevant bids at $t = 4$. A buyer who first bids in auction 1 will seriously bid in auction 2 at $t = 4$, only if he is supposed to bid in auction 1 at $t = 3$. Suppose such equilibrium exists. Let player i bid first $b(v_i)$ in auction 1 at $t = 3$. If he losses in auction 1, he has a weakly dominant strategy to bid v_i in auction 2 at $t = 4$. If he wins in auction 1, he weakly prefers not to bid any more. His expected payoff can be written as:

$$\Pi = \alpha\pi_1(b(.)) + \alpha(1 - \alpha)\pi_2$$

where $\pi_1(b(.))$ denotes the expected payoff of player i, when his bid is accepted in auction 1 and π_2 is the expected payoff of player i when his bid is rejected in auction 1 and accepted in auction 2. Since $b(v_i)$ is the equilibrium bidding function, player i has no incentives to deviate by bidding $b(v_i)$ in auction 1 at $t = 2$. Hence:

$$\pi_1(b(.)) \leq \Pi$$

or equivalently:

$$\pi_1(b(.)) \leq \alpha\pi_2 \tag{4.5}$$

Similarly, player i has no incentives to deviate by bidding v_i in auction 2 at $t = 3$. Hence:

$$\pi_2 \leq \Pi$$

or equivalently:

$$\frac{1-\alpha(1-\alpha)}{\alpha}\pi_2 \leq \pi_1(b(.)) \tag{4.6}$$

Subtracting (4.5) in (4.6) gives:

$$\frac{1-\alpha(1-\alpha)}{\alpha}\pi_2 \leq \alpha\pi_2$$

or equivalently:

$$1-\alpha \leq 0$$

Since I have assumed that $\alpha \in (0,1)$, the latter is impossible. Hence, I have found a contradiction.

Suppose there is some other equilibrium. Then, there must be at least one buyer bidding in auction 1 before $t = 3$. Let it be buyer 1. By assumption at least two bidders follow the same strategy. Hence, there must be also some other buyer bidding in auction 1 before $t = 3$. Let it be buyer 2. Suppose that buyer 1 and buyer 2 bid according to $b(v_i)$, where $b'(v_i) < v_i$ in auction 1 at $t < 3$. Then, the ordering of two valuation is revealed. Refer now to the proof of proposition 4.4. Then it immediately follows that whenever the ordering of at least two valuations is revealed, the resulting outcome is efficient or equivalent to outcome 2 as defined in proposition 4.4. To finish the proof, it is enough to show that outcome 2 cannot arise. In the proof of proposition 4.4, outcome 2 arises only if the ordering of the two valuations is revealed at $t = 2$. Suppose then that buyer 1 and buyer 2 bid according to $b(v_i)$, where $b'(v_i) < v_i$ in auction 1 at $t = 2$. Then, by assumption the winning buyer cannot bid in auction 2. If so he weakly prefers to bid the valuation in auction 1 at $t = 3$. Then, the losing bidder prefers to bid the valuation in auction 2 at $t = 3$. Buyer 3 realizes that $\max[v_1, v_2]$ will be submitted in auction 1 and $\min[v_1, v_2]$ will be submitted in auction 2. Hence, he prefers to bid the valuation in auction 2. All in all, the resulting outcome is efficient. By the revenue equivalence theorem, the expected prices are given by $E[v^{3:3}]$. □

Part 2

Regret motive in auctions

CHAPTER 5

Regret in auctions - theory

5.1. Introduction

Experiments on independent private value sealed bid auctions show that human subjects overbid as compared to standard (risk neutral) auction theory (see Kagel, 1995, for a survey). Moreover, they do not learn the "optimal" behavior (see for example Kagel, Harstad and Levin, 1987, Güth et al., 2003, and Harstad, 2000). In order to provide an explanation for the aggressive bidding in a first price auction, the constant relative risk aversion model (CRRAM) was developed (see for example Cox, Smith and Walker, 1988). Harrison (1989) criticized the evidence supporting CRRAM by noting that observed deviations from the equilibrium prediction under risk neutrality would only generate very small losses in expected payoffs. Hence, despite "statistically significant deviations in terms of bids" there were no "statistically significant deviations in terms of foregone payoff". This "flat-maximum critique" gave the source of a polemic discussion, which resulted in the conclusion that risk aversion could be one of the forces driving the aggressive bidding, but not necessarily the only one. Following this stream, Cox, Smith and Walker (1992) mentioned utility of winning as another source of overbidding. They suggested that a bidder might like winning and therefore bid higher. Alternatively, Goeree et al. (2002) showed that the Quantal Response model of risk averse bidders nicely fits their experimental data on overbidding in a first price sealed bid auction.

Overbidding in a second price sealed bid auction is more difficult to justify than the aggressive bidding in a first price sealed bid auction. A buyer who takes into account only the monetary payoff weakly prefers to bid the valuation, regardless of risk preferences. In order to understand the aggressive bidding, one should thus extend the existing theory by incorporating factors that most likely affect the behavior of experimental subjects[1]. Following this path, Morgan, Steiglitz and Reis (2003) proposed a model in which spiteful bidders compete against each other. They model spitefulness by assuming that loser's utility decreases in the winner's surplus. Realizing that high bids make others suffer, a player is ready to sacrifice a part of his expected monetary payoff to be compensated by the decreased expected payoff of all his opponents. As Morgan, Steiglitz and Reis (2003) noted, the spitefulness is a feeling that is experienced, when competing against people and not machines. Hence, when playing against computerized players, spiteful subjects should bid less. However, as Cox, Smith and Walker (1987) observed, the presence of non-human opponents does not trigger a shift in bidders' behavior. Thus, spitefulness rather does not drive overbidding.

The present paper explains how regret and rejoicing could rationalize overbidding. Regret describes a negative, cognitively based feeling that we experience, when realizing that the present situation would have been better, if we had decided differently (see Connolly and Zeelenberg (2002) as well as Gilovich and Medvec (1995) for overviews). Rejoicing is a positive counterpart of regret and refers to a reaction that appears after having chosen the action that is optimal in a given state of the world (see for example Landman (1993)). Regret and rejoicing are not only experienced, but also anticipated. The anticipated regret/rejoicing affects decision making (see for example Zeleenberg (1999)). Regret theory incorporates regret and rejoicing into the classical analysis of decision making under uncertainty. It assumes that a decision-maker maximizes (modified) expected utility, which implies minimizing expected regret and maximizing expected rejoicing (see Bell (1982) as well as Loomes and Sugden (1982)). Regret and rejoicing are closely related with information feedback. When given information about missed opportunities, we experience regret or rejoicing. If by contrast the outcomes of unchosen actions are not revealed, regret and rejoicing do not take place (see Camille et al. (2004) for an example).

[1]Clearly, incorporating utility of winning in the standard auction model is one of the possibilities.

Engelbrecht-Wiggans (1989) introduced regret into the context of first price auctions. He distinguished two types of regret in a first price sealed bid auction. First, an agent regrets, when losing at a price below the valuation. Second, a positive difference between his bid and the second highest bid makes a winner regret. His results show that when the two types of the regret are equally weighted, the classical solution is found. If the former type of the regret matters less (more) than the latter, a buyers bids less (more) aggressively. Furthermore, he suggested that regret of winning is more important than regret of losing and therefore a first price auction should yield lower revenue than predicted by the standard theory.

The present paper shows how, under different information feedbacks, regret and rejoicing affect optimal bidding in sealed bid auctions. As compared to Engelbrecht-Wiggans (1989), it allows for a wider family of regret functions, incorporates rejoicing and also studies a second price sealed bid auction. Furthermore, it mainly concentrates on situations in which bidder is more affected by regret of losing than by regret of winning. Finally, contrary to Engelbrecht-Wiggans (1989), it argues that a first price sealed bid auction should yield higher revenue than predicted by the standard theory. The proposed model explains the aggressive bidding observed in experimental sealed-bid auctions and predicts some of the regularities observed in experimental sealed bid auctions. It shows that the number of bidders has a positive impact on the equilibrium bidding function in a first price sealed bid auction. In a second price sealed bid auction, the equilibrium bidding function is independent of the number of bidders. An equilibrium price of a second price sealed bid auction is higher than the equilibrium price of the first price sealed bid auction. Finally, providing information about the price or hiding the value of the second highest bid increases auction's revenue.

The paper is structured as follows. The next section defines first- and second-price sealed bid auctions as games in which bidder's utility involves both monetary payoff and regret/rejoicing. Furthermore, it defines overbidding, underbidding and the classical outcome. Section 5.3 reports the results. It starts from studying the general case of a first price sealed bid auction. Then, it proceeds with studying the role of the linear regret function in optimal bidding in a first price sealed bid auction. Afterwards, it explains how regret and rejoicing lead to overbidding in a second price sealed bid auction and demonstrates implications of the linear regret function for a second price sealed bid auction. Section 5.4 discusses experimental results supporting the proposed model. Finally, section 5.5 concludes.

5.2. The Model

5.2.1. Preliminaries

There are $n \geq 2$ buyers competing in a single object auction. Each buyer i ($i = 1, ..., n$) has an independent private valuation of the object v_i, which is drawn from the increasing distribution function $F(v_i)$ with density $f(.) \equiv F'(.)$ and support on [0,1]. $F^{(1)}(.)$ and $f^{(1)}(.)$ define the cumulative distribution function and density function respectively of $v_{(1)} \equiv \max_{j \neq i} v_j$.

I study a *first price sealed bid auction* (FPSB) and a *second price sealed bid auction* (SPSB). In both of them each buyer i simultaneously submits a sealed bid ($b(v_i)$) at least equal zero. The one whose bid is the highest wins the auction. If several players submit the highest bid, which, in equilibrium, is a zero probability event, one of them is randomly selected to be the winner. The winner obtains the object at price p. The rest of the players obtain zero monetary payoff. In FPSB the price equals to the highest bid. In SPSB the final price is given by the second highest bid, or if all bids equal to each other, to the highest bid.

I consider different information feedbacks. The price is revealed to all players or only to the winner. The value of the second highest bid ($\bar{b}$) is or is not be publicly announced.

In each auction format and for each information feedback, the monetary payoff of buyer i is given by:

$$MP_i = \begin{cases} v_i - p & \text{if } i \text{ wins} \\ 0 & \text{otherwise} \end{cases}$$

5.2.2. Regret motive

Depending on the information feedback, each buyer appropriately transfers his monetary payoff into the utility function. The utility of buyer i (u_i) is given by:

$$u_i = \begin{cases} v_i - p - I_{\bar{b}} W(p - \bar{b}) & \text{if } i \text{ wins} \\ -I_p L(v_i - p) & \text{otherwise} \end{cases} \tag{5.1}$$

where:

- $\bar{b}$ denotes the value of the second highest bid,
- $I_{\bar{b}} = \begin{cases} 0 & \text{if } \bar{b} \text{ is not known to the winning bidder} \\ 1 & \text{otherwise} \end{cases}$
- $I_p = \begin{cases} 0 & \text{if } p \text{ is not revealed to the losing bidder} \\ 1 & \text{otherwise} \end{cases}$
- $W(p - \bar{b})$ describes the regret of bidding too high,
- $L(v_i - p)$ measures the regret of bidding too low and rejoicing of not bidding too high

If buyer i wins and does not learn $\bar{b}$, his utility is given by the monetary payoff. When he wins and learns $\bar{b}$, his utility equals the monetary payoff decreased by the regret of not overbidding the second highest bid by the smallest available increment ($W(.)$). $W(.)$ is continuos and nonnegative for every $p > \bar{b}$. A buyer never regrets, when winning at the price that exactly equals to the second highest bid, i.e. $W(0) = 0$. Since paying more is presumably at least as harmful as paying less, $W'(.) \geq 0$. In a second price sealed bid auction, $p = \bar{b}$ and hence winner's utility ($v_i - p - W(0)$) is only given by his monetary payoff ($v_i - p$).

If buyer i loses and does not learn p, his utility equals zero. If he loses and learns p, his emotions are measured by $L(.)$. I assume that $L'(.) \geq 0$ exists and is continuous. As a result, the losing bidder is more disappointed with the lower price. When the price is lower than the valuation, his utility is negative, i.e. $L(v_i - p) > 0$, if $v_i - p > 0$. I consider two specifications of $L(.)$:

- "Pure Regret": $L(v_i - p) = 0$, if $v_i - p \leq 0$
- "Regret/rejoicing": $L(v_i - p) < 0$, if $v_i - p < 0$

"Pure regret" implies that in a first price sealed bid auction, an agent regrets bidding too little, when he realizes that he would make a surplus, if he had overbid the highest bid by a very small number. In a second price sealed bid auction, a losing bidder does not know the exact value of the highest bid. When learning that the value of the second highest bid is below his valuation, he anticipates that the value of the highest bid is below his valuation with a positive probability. Hence, he realizes that if he had bid more, he would make a surplus with a positive probability and that is the source of his regret.

"Regret/rejoicing" adds rejoicing to the previous specification. In particular, a losing bidder regrets bidding too low, when the price is below the valuation, and rejoices not bidding too high, when the price is above the valuation. When the price is above the valuation, a losing bidder is sure that the highest bid is higher than his valuation in both FPSB and SPSB. He knows that if he had bid less, his situation would remained unchanged, and if he had bid more, he could lose money. As a result, he concludes that his choice of the bid was the best in the given state of the world and that causes his rejoicing.

5.2.3. Possible strategies

In the equilibrium of the standard (risk neutral) model of the first price sealed bid auction, a buyer bids:

$$b^*_{(1)}(v_i) = v_i - \frac{\int_0^{v_i} F(x)^{(n-1)} dx}{F(v_i)^{(n-1)}} \tag{5.2}$$

(see Krishna (2002)). In the equilibrium of the standard model of the second price sealed bid auction, a buyer bids:

$$b^*_{(2)}(v_i) = v_i \tag{5.3}$$

Let $\hat{b}_{(1)}(v_i)$ and $\hat{b}_{(2)}(v_i)$ to be a Bayesian Nash equilibrium strategy of FPSB and SPSB respectively. Buyer i overbids in FPSB (SPSB), whenever $\hat{b}_{(1)}(v_i) \geq b^*_{(1)}(v_i)$ $(\hat{b}_{(2)}(v_i) \geq v_i)$ for every $v_i \in [0,1]$ and $\hat{b}_{(1)}(v_i) > b^*_{(1)}(v_i)$ $(\hat{b}_{(1)}(v_i) > v_i)$ for some $v_i \in [0,1]$. FPSB (SPSB) leads to the classical outcome, when $\hat{b}_{(1)}(v_i) = b^*_{(1)}(v_i)$ $(\hat{b}_{(2)}(v_i) = v_i)$ for every $v_i \in [0,1]$. Underbidding occurs in all the other cases.

5.2.4. Comparison with Engelbrecht-Wiggans (1989)

Engelbrecht-Wiggans (1989) studied the role of regret in a first price sealed bid auction. In particular, he assumed that utility of buyer i is given by:

$$u_i = \begin{cases} \alpha_0(v_i - p) - \alpha_1(b(v_i) - \bar{b})) & \text{if } i \text{ wins} \\ \min[-\alpha_2(v_i - p), 0] & \text{otherwise} \end{cases}$$

and showed that: (1) if both regrets are equally weighted ($\alpha_1 = \alpha_2$), the classical outcome arises and (2) increasing (decreasing) the importance of regret of winning (α_1) leads to the less (more) aggressive bidding. He also suggested that regret of winning is more important than regret of losing[2].

The present paper studies a wider family of regret functions. I also allow a bidder to rejoice not bidding too high when the price is above his valuation. Furthermore, I study both a first- and a second-price sealed bid auction. Finally, I mainly concentrate on the case where regret of winning is less important than regret of losing[3].

[2]When the paper was published, the law required all bids to be made public in US mineral lease auctions. Therefore, for example Exxon knew that the second highest bid was only a few per cent of the many millions of dollars that it had paid for one particular oil lease. Engelbrecht-Wiggans (1989) claimed that those money left on the table was far more noticeable to stockholders and superiors than missed opportunities to have won the object at a favorable price.

[3]I concentrate mainly on cases where regret of losing is more important than regret of winning, because, in practise, many first price auctions do not reveal the value of the second highest bid and therefore avoid regret of winning. In section 4 provides the evidence that regret of losing is more important than regret of winning.

5.3. Results

5.3.1. First Price Sealed Bid Auction

5.3.1.1. General results. According to standard auction theory a risk-neutral buyer bids $b^*_{(1)}(v_i) = E[v_{(1)}|v_{(1)} < v_i]$ in an equilibrium (see Krishna (2002)). This way, each buyer assures that he will win the object and will not have to pay too much, whenever he has the highest valuation. The presence of regret changes the classical reasoning twofold. First, when bidding $b_{(1)}(v_i) = E[v_{(1)}|v_{(1)} < v_i]$, a buyer often loses at price lower than his valuation and that may make him regret bidding too little. Anticipating the future emotions, each agent may thus upwardly adjust the bid, so that losing is not so harmful. Second, when bidding more than the strongest opponent, a buyer may regret bidding too high. If being informed about the value of the highest bid of all competitors in the end, a buyer anticipates the future regret of winning and bids less. Depending on the information feedback, these two forces appropriately balance in an equilibrium.

Proposition 5.1. *In a symmetric differentiable pure Bayesian Nash equilibrium of FPSB, the following holds:*

- *if $I_{\bar{b}} = 0$ and $I_p = 0$, the classical outcome arises,*
- *if $I_{\bar{b}} = 0$ and $I_p = 1$, a buyer overbids and $\hat{b}_{(1)}(v_i) < v_i$ for every $v_i \in (0, 1]$,*
- *if $I_{\bar{b}} = 1$, $I_p = 0$ and $W'(0) > 0$, a buyer underbids.*

To conclude, in the absence of regret of winning, regret of losing leads to overbidding in both "pure regret" and "regret/rejoicing". More regret of losing induces more aggressive bidding. Since a winner has to pay what he bids, overbidding has its limit. In particular, a buyer always bids below the valuation. Emotions experienced after losing at the price exceeding one's valuation do not affect equilibrium bidding. In the presence of the regret of winning, the incentives to overbid are reduced.

5.3.1.2. Linear regret functions. To have a more detailed description of the equilibrium, I need to impose several assumptions. Suppose that:

$$I_{\bar{b}}W(p-\bar{b}) = \beta(p-\bar{b}) \tag{5.4}$$

$$I_pL(v_i-p) = \alpha(v_i-p) \tag{5.5}$$

where $\beta \in [0,1)$ and $\alpha \in [0,1)$. In this specification α measures the degree of regret/rejoicing of losing and β denotes the degree of regret of winning. Clearly, $\beta = 0$ corresponds to $I_{\bar{b}} = 0$ and $\alpha = 0$ to $I_p = 0$. Given this specification, the equilibrium can be defined as follows.

Proposition 5.2. *Suppose that (5.4) and (5.5) hold. Then, the unique symmetric differentiable pure Bayesian Nash equilibrium of FPSB is characterized by:*

$$\hat{b}_{(1)}(v_i) = E[v_{(1)}|v_{(1)} < v_i] + \left[\frac{\int_0^v F(t)^{(n-1)}dt}{F(v_i)^{(n-1)}} - \frac{\int_0^{v_i} F(x)^{\frac{(n-1)(1+\alpha)}{1+\beta}}dx}{F(v_i)^{\frac{(n-1)(1+\alpha)}{1+\beta}}}\right] \tag{5.6}$$

The optimal bidding function given by (5.6) depends on α and β. The comparison of these two parameters gives the prediction of the outcome resulting from the equilibrium.

Corollary 5.3. *Suppose that (5.4) and (5.5) hold. Then, the unique symmetric differentiable pure Bayesian Nash equilibrium of FPSB leads to:*

- *overbidding, iff* $\alpha > \beta$,
- *classical outcome, iff* $\alpha = \beta$,
- *underbidding, iff* $\alpha < \beta$.

The optimal bidding function also depends on the number of bidders. The appendix shows that increasing the number of the bidders leads to more aggressive bidding. This result is not surprising, as in the standard setting, without any type of regret, the optimal bidding function is also increasing in n.

Corollary 5.4. *Suppose that (5.4) and (5.5) hold. Then, in the Bayesian Nash equilibrium of FPSB the optimal bidding function is increasing in the number of bidders*

In most of the experimental settings, uniform distribution is used. Given this distribution, the optimal bidding function is given as follows.

Corollary 5.5. *Suppose that (5.4) and (5.5) hold. Then, for the uniform distribution the equilibrium bidding function simplifies to:*

$$\hat{b}_{(1)}(v_i) = v_i \left(1 - \frac{1+\beta}{n + \alpha(n-1) + \beta}\right) \tag{5.7}$$

For the uniform distribution, the optimal bidding function is increasing in the number of bidders, increasing in α, decreasing in β and strictly smaller than v_i. Hence, more aggressive bidding comes from bidders facing more competitors or more regretful ones. Even the most regretful subject with the very high number of opponents does not bid above the valuation.

5.3.2. Second price sealed bid auction

5.3.2.1. General results. In the standard setting of a second price sealed bid auction, it is a weakly dominant strategy to bid the valuation (see Krishna (2002)). Bidding less leads to a risk of losing a positive transaction; bidding more increases chances of winning the object but at an unprofitable price. In our model, equilibrium behavior is affected by the regret/rejoicing of losing; regret of winning plays no role. Bidding below the valuation is still weakly dominated by bidding the valuation. In "pure regret", bidding above the valuation is also dominated by bidding the valuation. In "regret/rejoicing", however, bidding the valuation is not necessarily a dominant strategy. By bidding more aggressively, an agent increases the expected price. Higher expected price implies higher expected rejoicing when losing. As a result, overbidding may arise in an equilibrium, which we show in the next proposition.

Proposition 5.6. *A symmetric differentiable pure Bayesian Nash equilibrium of SPSB leads to:*

- *classical outcome in the case of "pure regret",*
- *overbidding or the classical outcome in the case of "regret/rejoicing".*

The proposition implies that overbidding may occur in many specifications of the regret function. In the next subsection we study the implications of the linear regret function on optimal bidding behavior.

5.3.2.2. Linear regret function. The present subsection studies equilibrium characteristics for:

$$I_p L(v_i - p) = \alpha(v_i - p) \tag{5.8}$$

where $\alpha \in (0, 1)$. The next proposition characterizes the equilibrium.

Proposition 5.7. *Suppose that (5.8) holds. Then, the unique symmetric differentiable pure Bayesian Nash equilibrium of SPSB is characterized by:*

$$\hat{b}_{(2)}(v_i) = v_i + \frac{\int_{v_i}^{1}(1 - F(t))^{(1+\alpha)/\alpha}dt}{(1 - F(v_i))^{(1+\alpha)/\alpha}} \tag{5.9}$$

Since (5.9) equals to the optimal bidding function of a second price sealed bid auction derived by Morgan et al. (2002)[4] and (5.6) is lower or equal to their optimal bidding function of a first price sealed bid auction, we use some of their findings:

Corollary 5.8. *Suppose that (5.8) holds. Then, in the Bayesian Nash equilibrium a second price sealed bid auction yields higher revenue than a first price sealed bid auction.*

[4] Morgan, Steiglitz and Reis (2002) model the spiteful motive by means of the following utility function:

$$u_i = \begin{cases} v_i - p & \text{if } i \text{ wins} \\ -\alpha(v_j - p) & \text{if } j \neq i \text{ wins} \end{cases} \qquad \text{where } \alpha \in (0, 1)$$

Although our model fundamentally differs from their approach, the two models sometimes produce the same predictions (e.g. when (5.8) holds in a second price sealed bid auction). The two models would give different predictions for example for the first price sealed bid auction in which the raised revenue is equally divided between all losers.

Furthermore, in the Bayesian Nash Equilibrium of a second price sealed bid auction, the following holds:

- $\frac{\partial \hat{b}_{(2)}(v_i)}{\partial n} = 0,$
- $\frac{\partial \hat{b}_{(2)}(v_i)}{\partial \alpha} > 0,$
- *if* $F(v_i) = v_i$, *then* $\hat{b}_{(2)}(v_i) = \left(\frac{1+\alpha}{1+2\alpha}\right) v_i + \frac{\alpha}{1+2\alpha}$,

  for every $v_i \in [0, 1]$

We conclude that first, our model predicts that a second price sealed bid auction yields higher revenue than a first price sealed bid auction. Second, equilibrium bidding strategies are independent of the number of bidders. Third, similarly as in a first price sealed bid auction, people that are more emotional when losing (those with higher α), bid more aggressively. Finally, for the uniform distribution, the optimal bidding function is proportional to the valuation and includes a constant term.

5.4. Experimental evidence

Our model explains the overbidding that was found in experiments on sealed bid auctions. It also predicts that a second price sealed bid auction yields higher revenue than a first price sealed bid auction, just as observed in several experiments (see Kagel (1995)). Furthermore, increasing the number of bidders was shown to lead to more aggressive bids in a first price sealed bid auction and to have no effect on bidding in a second price sealed bid auction, which was experimentally found by Kagel and Levin (1993). Moreover, our model indicates that for the uniform distribution, which is used in most experiments (compare Kagel (1995)), the optimal bidding function does not include a constant term and is proportional to one's valuation in a first price sealed bid auction, as experimentally observed by Holt and Sherman (2000). In a second price sealed bid auction, for the uniform distribution, the optimal bidding function is proportional to the valuation and includes the constant term, which was present in the experimental results of Kagel and Levin (1993).

The relation between regret/rejoicing and overbidding seems to be apparent in the experiments run by Ockenfels and Selten (2005) as well as Neugebauer and Selten (2006). Both studies tested the presence of learning in a first price sealed bid auction by varying the information feedback. In the first experiment, two information feedbacks (*NF* and *F*) were tested. A subject competed with each time randomly matched opponent. His valuation was drawn according to the uniform distribution from the interval [0,100]. After learning the valuation, he was asked to submit the bid. If his bid was higher than the one of the opponent, he won the auction. If his bid was lower than the bid of the opponent, he lost. In case of a tie, the winner was chosen randomly. After submitting the bid, each bidder was informed whether he won the auction, the price and his payment for the auction. Under treatment *NF*, no additional feedback was given. Under *F*, the winner was informed about the bid of his opponent. The two treatments stimulated different bidding behavior. The overbidding that was observed in *F* was significantly reduced in *NF*.

In the experiment of Neugebauer and Selten (2006), having the same resale value, a subject competed with randomly matched computerized bids[5] in 100 consecutive first price sealed bid auctions. The valuation of a given subject equalled the upper bound of the uniform distribution from which the $N-1$ competitors' bids[6] were identically and independently drawn[7]. Three different treatments (T0, T1 and T2) were considered. In treatment T0 no information about the highest bid of the competitors was provided. In treatment T1, as usual, a subject was informed about the highest bid of the competitors only after losing. In treatment T2 the highest bid of competitors was always revealed.

If subjects' had behaved in accordance with the prediction of classical (risk neutral or risk averse) auction theory, the three conditions would have not induced differing behavioral patterns. However, the overbidding was highly reduced in T0 and T2. In

[5]As Cox, Smith and Walker (1987) report, the presence of non-human competitors does not significantly affect bidding behavior in a first price sealed bid auction.

[6]N was 3, 4, 5, 6 or 9 depending on the treatment.

[7]Under these conditions, a risk neutral buyer submits a bid as in a standard first price sealed bid auction and a risk-averse buyer bids as in a constant relative risk aversion model of a first price sealed bid auction.

particular, on average 75% of all subjects could be identified as overbidding in T1. The corresponding numbers for T0 and T2 amounted to 41% and 55% respectively.

The directions of changes of bids can be explained by our model. In T1 and *NF* the loser knows what he would obtain, if he were a winner. Hence, anticipating future regret, he overbids. In T0 the loser is not able to calculate the foregone payoff and hence does not regret losing as much as in T1. Therefore, observed bids should be lower than in T1. In T2 and *F* an agent has enough information to regret both losing and winning. Therefore, he should bid less than in T1 and *F*.

Finally, it is interesting to note that the experimental results of Neugebauer and Selten (2006) suggest that regret of winning has a weaker effect than regret of losing. While comparing T1 with T2 we can deduce that the presence of regret of winning reduced the overbidding from the level of 75% to 55%. On the other hand, when confronting T1 and T0 we can infer that reducing the regret of losing causes a decline in the overbidding from 75% to 41%. Hence, although both regret of winning and regret of losing seem to affect bidding behavior, the effect of regret of losing is presumably more important. This supports our approach, in which we pay more attention to regret of losing than to regret of winning, and is opposite to the approach of Engelbrecht-Wiggans (1989), who suggested that regret of winning is more important than regret of losing.

5.5. Conclusions

The present paper shows that anticipated emotions could cause the aggressive bidding observed in experiments. Our model enriches the classical auction model by claiming that depending on the information feedback a loser is exposed to several emotions that have been investigated by psychologists. Having no relevant information, the loser does not experience any emotions. Learning the value of the price, the loser does not remain calm, but reacts emotionally. He regrets having bid too low or rejoices not having bid too much. The information about the value of the second bid makes him sad in a first price sealed bid auction, because he does not like having to pay too much.

In the presence of feedback about the price, incentives to overbid occur. Agents bid high because of the anticipated negative emotions (regret of losing a chance for a positive transaction) or due to anticipated positive emotions (rejoicing of not having to overpay). The presence of the information about the second highest bid causes negative emotions (regret of overpaying), which reduces the aggressive bidding.

One straightforward implication of our model is that reducing the availability of the information about the price should reduce the overbidding. Furthermore, providing data about the second highest bid should lead to less aggressive bidding in a first price sealed bid auction. Both predictions have been found to be valid by Neugebauer and Selten (2006).

Regret and rejoicing could have also further effects on the bidding behavior. Since regret of action is known to be a temporary pattern (see Gilovich and Medvec (1995)), overbidding should decrease, when subjects are given more time to take the cognitive effort of reducing their emotions. That's perhaps why, subjects do not tend to overbid in open auction formats.

All in all, regret and rejoicing seem to affect experimental subjects. To make starker conclusions, more research is needed. First, it should be emphasized that the supporting experimental evidence is also well explained by the learning model described by Ockenfels and Selten (2005) as well as Neugebauer and Selten (2006). It is desirable to experimentally distinguish between these two underlying motives of the bidders behavior. Second, the proposed model is static. Bidders maximize their expected payoff to derive an optimal bidding function that does not depend on the experienced emotions. In the reality, not only the anticipated emotions matter, but also the experienced emotions affect the behavior. Hence, there is a need for a dynamic model of the bidding behavior incorporating both anticipated and experienced emotions.

5.6. Appendix

Proof. (proposition 5.1) A standard argument implies that a symmetric equilibrium bidding function $(\hat{b}_{(1)}(v_i))$ must be strictly increasing and continuous. Suppose that every player $j \neq i$ bids $b_{(1)}(v_j)$ and that player i bids $b_{(1)}(y)$ that maximizes his expected payoff:

$$\int_0^y (v_i - b_{(1)}(y) - I_{\bar{b}} W(b_{(1)}(y) - b_{(1)}(v_j))) f^{(1)}(v_j) dv_j - \int_y^1 I_p L(v_i - b_{(1)}(v_j)) f^{(1)}(v_j) dv_j$$

The first term is the expected utility in case of winning and the second term describes the expected regret of losing at the price $b_{(1)}(v_{(1)})$. The first derivative of the above expression simplifies to:

$$(v_i - b_{(1)}(y) + I_p L(v_i - b_{(1)}(y))) f^{(1)}(y) - b'_{(1)}(v_i) F^{(1)}(y)(1 + I_{\bar{b}} W'(0))$$

Since in equilibrium $y = v_i$:

$$(v_i - b_{(1)}(v_i) + I_p L(v_i - b_{(1)}(v_i))) f^{(1)}(v_i) - b'_{(1)}(v_i) F^{(1)}(v_i)(1 + I_{\bar{b}} W'(0)) \equiv \Gamma(b_{(1)}(v_i))$$

Now, it is straightforward to see that a buyer with $v_i = 0$ bids 0 in an equilibrium (i.e. $\hat{b}_{(1)}(0) = 0$). Therefore, we concentrate on $v_i \in (0, 1]$ for the rest of the proof. Note that: $v_i - b_{(1)}(v_i) + I_p L(v_i - b_{(1)}(v_i)) \equiv \Theta > 0$ for $b_{(1)}(v_i) < v_i$, $\Theta \leq 0$ for $b_{(1)}(v_i) \geq v_i$, $f^{(1)}(v_i) \geq 0$, $F^{(1)}(v_i) > 0$, $I_{\bar{b}} W'(0) \geq 0$ and $b'_{(1)}(v_i) > 0$. Hence, $\Gamma(b_{(1)}(v_i)) < 0$ for $b_{(1)}(v_i) \geq v_i$. Therefore, optimal $b_{(1)}(v_i)$ $(\hat{b}_{(1)}(v_i))$ must be strictly lower than v_i (i.e. $\hat{b}_{(1)}(v_i) < v_i$ for every $v_i \in (0, 1]$).

Note that in equilibrium $\Gamma(\hat{b}_{(1)}(v_i)) = 0$. Hence:

(5.10)

$$\Psi(v_i + I_p L(v_i - \hat{b}_{(1)}(v_i))) f^{(1)}(v_i)(F^{(1)}(v_i))^{\Psi-1} = \Psi \hat{b}_{(1)}(v_i) f^{(1)}(v_i)(F^{(1)}(v_i))^{\Psi-1} + \hat{b}'_{(1)}(v_i)(F^{(1)}(v_i))^{\Psi}$$

where $\Psi \equiv \frac{1}{1+I_{\bar{b}}W'(0)}$. Integrating both sides yields:

$$\hat{b}_{(1)}(v_i) = \frac{\Psi}{(F^{(1)}(v_i))^{\Psi}} \int_0^{v_i} (x + I_p L(x - \hat{b}_{(1)}(x))) f^{(1)}(x)(F^{(1)}(x))^{\Psi - 1} dx \tag{5.11}$$

where the condition that $\hat{b}_{(1)}(0)$ is finite is used to obtain the constant from integration. Integrating by parts gives:

(5.12)

$$\hat{b}_{(1)}(v_i) = v_i + I_p L(v_i - \hat{b}_{(1)}(v_i)) - \frac{1}{(F^{(1)}(v_i))^{\Psi}} \int_0^{v_i} (1 + I_p L'(x - \hat{b}_{(1)}(x))(1 - \hat{b}'_{(1)}(x)))(F^{(1)}(x))^{\Psi} dx$$

After noting that $\frac{\partial \Psi}{\partial \theta} = -I_{\bar{b}}\Psi^2$, where $\theta \equiv W'(0)$, one differentiates (5.12) with respect to θ to find:

$$\frac{\partial \hat{b}_{(1)}(v_i)}{\partial \theta} = -I_{\bar{b}}\Psi^2 \left(-\frac{1}{(F^{(1)}(v_i))^{\Psi}} \int_0^{v_i} \Omega(x, \Psi)(\ln\left[F^{(1)}(x)\right] - \ln\left[F^{(1)}(v_i)\right]) dx \right)$$

where $\Omega(x, \Psi) \equiv (1 + I_p L'(x - \hat{b}_{(1)}(x))(1 - \hat{b}'_{(1)}(x)))(F^{(1)}(x))^{\Psi}$. Note that: $L'(x - \hat{b}_{(1)}(x)) \geq 0$, $F^{(1)}(x) > 0$ and $\hat{b}'_{(1)}(x) < 1$. Therefore, $\frac{\partial \hat{b}_{(1)}(v_i)}{\partial \theta} < 0$, if $I_{\bar{b}} \neq 0$, and $\frac{\partial \hat{b}_{(1)}(v_i)}{\partial \theta} = 0$, otherwise.

Suppose now that $W'(0) = 0$ or $I_{\bar{b}} = 0$. Rewrite (5.11) as:

$$\hat{b}_{(1)}(v_i) = b^*_{(1)}(v_i) + \frac{I_p}{F^{(1)}(v_i)} \int_0^{v_i} L(x - \hat{b}_{(1)}(x)) f^{(1)}(x) dx$$

where $b^*_{(1)}(v_i)$ defines the solution that would be found, if $L(x - \hat{b}_{(1)}(x)) = 0$ for every $x \in [0, v_i]$ or $I_p = 0$. As already argued, $\hat{b}_{(1)}(v_i) < v_i$. Hence, if $I_p = 1$, $L(x - \hat{b}_{(1)}(x)) > 0$ for every $x \in [0, v_i]$ and, as a result, $\hat{b}_{(1)}(v_i) > b^*_{(1)}(v_i)$. □

Proof. (proposition 5.2) Suppose that (5.4) and (5.5) hold. Then, after rearranging, (5.10) becomes:

$$\frac{1+\alpha}{1+\beta} v_i f^{(1)}(v_i) F^{(1)}(v_i)^{\frac{\alpha-\beta}{1+\beta}} = \frac{1+\alpha}{1+\beta} \hat{b}_{(1)}(v_i) f^{(1)}(v_i) F^{(1)}(v_i)^{\frac{\alpha-\beta}{1+\beta}} + \hat{b}'_{(1)}(v_i) F^{(1)}(v_i)^{\frac{1+\alpha}{1+\beta}}$$

Integrating both sides gives:

$$\hat{b}_{(1)}(v_i) = \frac{1+\alpha}{(1+\beta)F^{(1)}(v_i)} \int_0^{v_i} x f^{(1)}(x) \left(\frac{F^{(1)}(x)}{F^{(1)}(v_i)} \right)^{\frac{\alpha-\beta}{1+\beta}} dx$$

where the condition that $b_{(1)}(0)$ is finite is used to obtain the constant from integration. Now, using the integration by parts and $F^{(1)}(v_i) = F(v_i)^{(n-1)}$, one writes the above expression as:

$$\hat{b}_{(1)}(v_i) = v_i - \frac{\int_0^{v_i} F(x)^{\frac{(n-1)(1+\alpha)}{1+\beta}} dx}{F(v_i)^{\frac{(n-1)(1+\alpha)}{1+\beta}}} \tag{5.13}$$

or equivalently as (5.6). □

In order to check the effect of the number of bidders on the optimal bidding function we differentiate (5.13) with respect to n:

$$\begin{aligned} \frac{\partial \hat{b}_{(1)}(v_i)}{\partial n} &= -\frac{1+\alpha}{1+\beta} \int_0^{v_i} \frac{F(x)^{\frac{(n-1)(1+\alpha)}{1+\beta}}}{F(v_i)^{\frac{(n-1)(1+\alpha)}{1+\beta}}} \cdot (\ln F(x) - \ln F(v_i)) dx \\ &> 0 \end{aligned}$$

where the fact that $\alpha \geq 0$, $\beta \geq 0$, $n > 1$ and $F(x) > 0$ for every $x > 0$ was used to show the inequality.

Proof. (proposition 5.3) As in the proof of proposition 5.1, a symmetric equilibrium bidding function must be strictly increasing and continuous. Note that $I_p = 0$ describes the classical setting, in which it is optimal to bid the valuation. Suppose now that $I_p = 1$, every player $j \neq i$ bids $b_{(2)}(v_j)$ and that player i bids $b_{(2)}(y)$ that maximizes the expected payoff:

$$\int_0^y \left(v_i - b_{(2)}(v_j)\right) f^{(1)}(v_j) dv_j - \pi(y) L(v_i - b_{(2)}(y)) - \int_y^1 (L(v_i - b_{(2)}(v_j))) dF^{(2)}(v_j)$$

where $\pi(y)$ denotes the probability that there is exactly one opponent with a valuation larger than y and $F^{(2)}(v_j)$ denotes the distribution function of the second highest valuation of $n-1$ buyers. The first term of the expected payoff corresponds to the event in which player i wins an object at the price equal to $b_{(2)}(v_{(1)})$, the second term is associated with the event in which player i loses at the price equal to his bid and finally the last term is related to the event in which player i loses by bidding below the second highest bid.

The first derivative looks as follows:

$$(v_i - b_{(2)}(y))f^{(1)}(y) - \pi'(y)L(v_i - b_{(2)}(y)) + \pi(y)L'(v_i - b_{(2)}(y))b'_{(2)}(y) + L(v_i - b_{(2)}(y))f^{(2)}(y)$$

After rearranging, using the equilibrium condition $y = v_i$ and noting that $f^{(2)}(v_i) = f^{(1)}(v_i) + \pi'(v_i)$, one finds:

$$((v_i - b_{(2)}(v_i)) + L(v_i - b_{(2)}(v_i)))f^{(1)}(v_i) + \pi(y)b'_{(2)}(v_i)L'(v_i - b_{(2)}(v_i)) \equiv \Gamma(b_{(2)}(v_i))$$

Introduce:

$$\Omega(b_{(2)}(v_i)) \equiv ((v_i - b_{(2)}(v_i)) + L(v_i - b_{(2)}(v_i)))f^{(1)}(v_i)$$

$$\Phi(b_{(2)}(v_i)) \equiv \pi(y)b'_{(2)}(v_i)L'(v_i - b_{(2)}(v_i))$$

Note that by definition:

$$\Gamma(b_{(2)}(v_i)) = \Omega(b_{(2)}(v_i)) + \Phi(b_{(2)}(v_i))$$

Furthermore, $\Phi(b_{(2)}(v_i)) \geq 0$, $\Omega(b_{(2)}(v_i)) > 0$ for $b_{(2)}(v_i) < v_i$ and $\Omega(b_{(2)}(v_i)) < 0$ for $b_{(2)}(v_i) > v_i$. Hence, $\Gamma(b_{(2)}(v_i)) > 0$ for each $b_{(2)}(v_i) < v_i$. On the other side, $\Gamma(\infty) = -\infty$. Hence, the continuity of $\Gamma(b_{(2)}(v_i))$ implies $\hat{b}_{(2)}(v_i) \geq v_i$. Observe that when $L(0) = 0$ and $L'(0) = 0$ (that is sometimes in "regret/rejoicing" and always in "pure regret"), $\Gamma(v_i) = 0$. Since, $\Gamma(b_{(2)}(v_i)) < 0$ for $b_{(2)}(v_i) > v_i$ in "pure regret", the other equilibrium candidate does not exist for "pure regret". Hence, bidding one's valuation becomes an equilibrium strategy for all special cases of "pure regret" and some special cases of "regret/rejoicing". Now, assume that $L'(0) \neq 0$ (which is sometimes true in "regret/rejoicing"). Then, $\Gamma(b_{(2)}(v_i)) > 0$ for each $b_{(2)}(v_i) \leqslant v_i$ and hence $\hat{b}_{(2)}(v_i)$ has to be higher than v_i. □

Proof. (proposition 5.4) Substituting (5.8), $f^{(1)}(v_i) = (n-1)F(v_i)^{(n-2)}f(v_i)$ and $\pi(v_i) = (n-1)F(v_i)^{(n-2)}(1-F(v_i))$ in $\Gamma(\hat{b}_{(2)}(v_i)) = 0$ yields:

$$-\frac{1+\alpha}{\alpha}f(v_i)(1-F(v_i))^{\frac{1}{\alpha}}v_i = -\frac{1+\alpha}{\alpha}f(v_i)(1-F(v_i))^{\frac{1}{\alpha}}\hat{b}_{(2)}(v_i) + \hat{b}'_{(2)}(v_i)(1-F(v_i))^{\frac{1+\alpha}{\alpha}}$$

Integrating both sides gives:

$$\hat{b}_{(2)}(v_i) = C - \frac{1}{(1-F(v_i))}\int_0^{v_i}\frac{1+\alpha}{\alpha}xf(x)\left(\frac{1-F(x)}{1-F(v_i)}\right)^{\frac{1}{\alpha}}dx$$

Since $\hat{b}_{(2)}(1)$ is finite:

$$\hat{b}_{(2)}(v_i) = \frac{1}{(1-F(v_i))}\int_{v_i}^{1}\frac{1+\alpha}{\alpha}xf(x)\left(\frac{1-F(x)}{1-F(v_i)}\right)^{\frac{1}{\alpha}}dx$$

Now, using integration by parts, one writes the above expression as (5.9). □

CHAPTER 6

Regret in auctions - experiment

6.1. Introduction

There is considerable diversity in information feedback following submission of sealed bids in the field. Some of the auctioneers reveal all bids. Others only indicate the winner, so that non-winners have only the (unverified) information that their bids were lower. According to the standard auction theory, differences in the information feedback do not affect the optimal bidding behavior. In the reality, different information feedback may trigger different emotions (e.g. regret, spite, rejoicing) and as a consequence, different bidding patterns. This paper presents results of an experiment which varied the information in a first-price sealed-bid auction and in a second-price sealed-bid auction and discusses the effect of regret on bidders' behavior.

Different models of regretful bidders were proposed in the literature (see chapter 5 of this thesis, Engelbrecht-Wiggans (1989) and Engelbrecht-Wiggans and Katok (2006)). In the basic setting, a bidder maximizes his expected utility to derive an optimal bidding function. The utility incorporates anticipated regret depending on the information feedback. In a first price auction, a bidder realizes that if he loses and learns that the price is below his valuation, he will regret bidding too little. He also knows that if he learns that he has overpaid, he will regret bidding too high. On the other side, if the losing bidder does not learn the price, he will not be much concerned of bidding too little. Similarly, if the winning bidder does not know the missed opportunity, he will not think much about bidding too high. In equilibrium, all these forces balance so that expected feedback on the value of the price triggers more aggressive behavior and expected feedback on the value of the second highest bid results in less aggressive bidding.

There is experimental evidence supporting regret motive in first price auctions. Feedback on the second highest bid makes bidders less aggressive (see Isaac and Walker, 1985, Engelbrecht-Wiggans and Katok, 2006, Neugebauer and Selten, 2006 and Ockenfels and Selten, 2005). Informing losing bidders about the price leads to more aggressive bidding (see Neugebauer and Selten, 2006).

This paper studies regret motive in first price auctions and second price auctions. I experimentally study the relation between feedback on the price and bidding behavior. In contrast to Neugebauer and Selten (2006), I observe that providing feedback on the price lowers the average bids in a fist price auction. I also find that bidders' behavior is affected by experience of regret, which is strongly related to the information feedback.

The paper is structured as follows. The next presents an experimental design. The section 6.3 discusses the experimental results. Finally, the section 6.4 concludes.

6.2. Experimental design

The experiment was run in October 2004. The instructions are provided in the appendix. There were four treatments: $SPnI$, SPI, $FPnI$ and FPI, where the first two letters denote the type of the auction: second price (SP) or first price (FP) and the last letters indicate whether losers learn the price (I) or do not (nI). Each treatment was scheduled in one session. Each session had 35 rounds, including 5 practise rounds. The practise rounds took place at the beginning of the experiment to let subjects understand the design. During these rounds subjects did not earn money. In each session on average 19 students of Tilburg University were employed. Each student receives 2€ of show-up fee and the money earned in the experiment. The average earning was 9.15€. Each session took about one hour. The experiment was programmed and conducted with the software z-Tree (Fischbacher, 1999).

At the beginning of each round valuations were independently and uniformly drawn from $\{0, 1, ..., 400\}$, where 1 unit equaled 1 eurocent. No artificial currency was introduced to reduce confusion among bidders. Each subject was faced against two computerized bidders[1]. Subject were informed that computerized bidders bid as to maximize their expected payoffs, assuming that everybody else does the same.

After learning the valuation, a subject was asked to submit a bid. After placing the bid, a subject learned whether he had won or not. In case of winning, he also learned the price. In case of losing, he learned the price in SPI and FPI, but not in $SPnI$ and $FPnI$.

The payoff of the winner was given by his valuation decreased by the price, equal to the second highest bid in a second price sealed bid auction and to the highest bid in a first price sealed bid auction. The losing bidder received zero payoff.

6.3. Results

6.3.1. First-price sealed-bid auction

Data generated from this experiment include bids, prices and profits. Figure 6.1 depicts the average relative bids in the trial periods (from period -4 to period 0) and in the paid periods (from period 1 to period 30) for the treatment FPI and the treatment $FPnI$, as well as the average bids predicted by risk neutral Nash equilibrium of a standard model of a first price sealed bid auction ($RNNE$). The relative bid of a given subject is calculated by dividing his actual bid by his valuation. The average relative bid in a given period is the average of the relative bids submitted by all the subject in a given period under the same period. In risk neutral Nash equilibrium of a standard model of a first price sealed bid auction ($RNNE$), the relative bid is $\frac{2}{3}$ regardless of the information feedback.

Figure 6.1 shows that in most of the paid periods[2] the average relative bids under the treatment FPnI are higher and less volatile than the average bids under the treatment FPI. The Mann-Whitney U Test applied to subjects' average relative bids rejects the null hypothesis of equal average relative bids at the one percent significance level. Under both treatments, the actual relative bids are significantly higher than the RNNE predictions at

[1]Other studies have shown that in a first-price sealed-bid private-value auction, experimental subjects do not change their behavior when faced against computerized bidders instead of human bidders (see Cox et al., 1987, and Engelbrecht-Wiggans and Katok, 2006).

[2]Since the trial periods do not provide any incentives for the bidders, I focus only on the paid periods.

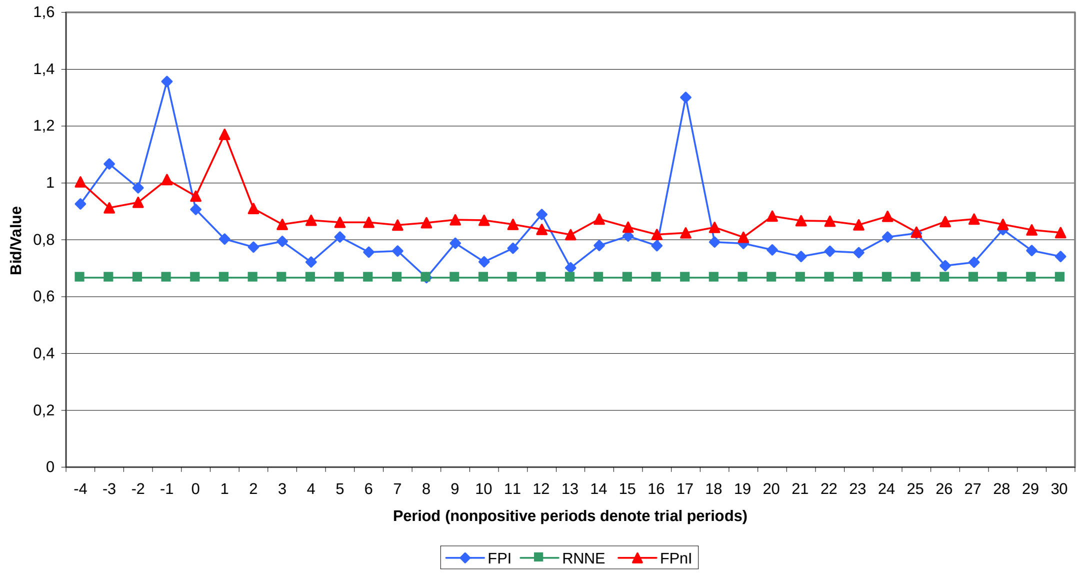

Figure 6.1. Average relative bids in FPI and FPnI

the one percent significance level. In other words, when given more information, subjects bid less aggressively but still above the RNNE predictions. Observed overbidding is common in the literature (see Kagel, 1995). The negative relation between the amount of the information and the bids suggests that risk aversion is not the main motive of the subjects. A risk-averse agent maximizes the utility over the expected payoff, where both the utility and the expected payoff do not depend on the information feedback. The observed relation is also contrary to the regret theory and the results of Neugebauer and Selten (2006). However, it is not clear that subjects reached an equilibrium. They might have needed more time. In the experiment of Neugebauer and Selten (2006), subjects participated in 100 auctions and had each time the same resale values, which made it much easier to discover the optimal bidding behavior.

There are three situations affecting the dynamic behavior:

- *money left on the table*: the subject wins the good at a price below his valuation,
- *missed opportunity to win*: the subject loses at a price below his valuation,
- *losing at an unprofitable price*: the subject loses at a price higher or equal to his valuation.

Treatment	Experience condition	bid increase # (%)	no change # (%)	bid decrease # (%)	row total
FPI	Money left on the table	113 (49%)	0 (0%)	116 (51%)	229 (40%)
	Missed opportunity to win	68 (88%)	0 (0%)	9 (12%)	77 (13%)
	Losing at an unprofitable price	113 (42%)	3 (1%)	156 (57%)	272 (47%)
	Total	294 (50%)	3 (1%)	281 (49%)	578
FPnI	Money left on the table	101 (40%)	1 (0%)	153 (60%)	255 (49%)
	Missed opportunity to win	31 (76%)	0 (0%)	10 (24%)	41 (8%)
	Losing at an unprofitable price	114 (51%)	2 (1%)	108 (48%)	224 (43%)
	Total	246 (47%)	3 (1%)	271 (52%)	520
Grand total		721 (49%)	6 (1%)	677 (50%)	1404

Figure 6.2. Bidders' reactions to different experiences in FPI and FPnI.

When there are "money left on the table", the winning bidder realizes that he could have won at a lower price. When there is "missed opportunity to win", the losing bidder realizes that he could have won at a profitable price. When "losing at an unprofitable price", the subject realizes that he could not improve his situation in any way.

Figure 6.2 depicts subjects' reactions to "money left on the table", "missed opportunity to win" and "losing at an unprofitable price". The reaction of a given bidder is counted as a difference between his current relative bid and the relative bid from the preceding period. When the difference is positive, the reaction is counted as "bid increase". No difference is interpreted as "no change". Negative difference is considered as "bid decrease". The experienced conditions are taken for the preceding period. A subject falls into "money left on the table" in the current period, if in the preceding periods he won the good. He experiences "missed opportunity to win", if he has just lost at the price below his valuation. "Losing at an unprofitable price" means that a subject has just lost at the price at least given by his valuation.

As figure 6.2 shows, subjects change their bids a lot. Among all the observed bidders reactions only 1% corresponds to "no change". On average, a subject is as likely to increase as to decrease his bid. The reaction depends on the experienced situation. Under "missed opportunity to win", a subject rather increases his bid and the reaction is stronger when he is given more information. This behavior could be experienced by regret motive. After losing at a price below the valuation, a subject regrets bidding too little and hence bids more aggressively in the next auction. Having less information, he is less conscious about regret of bidding too little and hence, is less aggressive in the next round[3].

The observed changes in the other experienced conditions are less structured. When there are "money left on the table", around half of the subjects increase their bids and about a half of the subjects decrease their bids. This behavior is not against the regret motive, as a subject was not provided feedback on the second highest bid and hence, was unable to calculate the foregone payoff and react to it. When losing at the price above the valuation, about a half of the subjects increase their bids and about a half of the subjects decrease their bids. This behavior is also not against the regret motive, as there was no space for possible improvement of bidder's situation.

Figure 6.3 summarizes the average reactions of the bidders. The differences in the reactions are even more striking than in figure 6.2. Being given enough information about the missed opportunity to win, a subject increases his bid by 12% on average. A less informed subject has difficulties in distinguishing "missed opportunity to win" from "losing at an unprofitable price". After losing at a profitable price, he increases his bid but by the smaller amount than a better informed subject. He also increases (by only 1%) his bid after losing at an unprofitable price. Hence, it seems that he is quite good at predicting the price, but makes some mistakes.

[3]The presented reactions resulting from the regret motive are similar to the reactions resulting from impulse balance theory. An interested reader is referred to Neugebauer and Selten (2006) and Ockenfels and Selten (2005).

	FPI	FPnI
Money left on the table	5%	-4%
Missed opportunity to win	12%	4%
Losing at an unprofitable price	-8%	1%

Figure 6.3. Average reactions of the bidders in FPI and FPnI

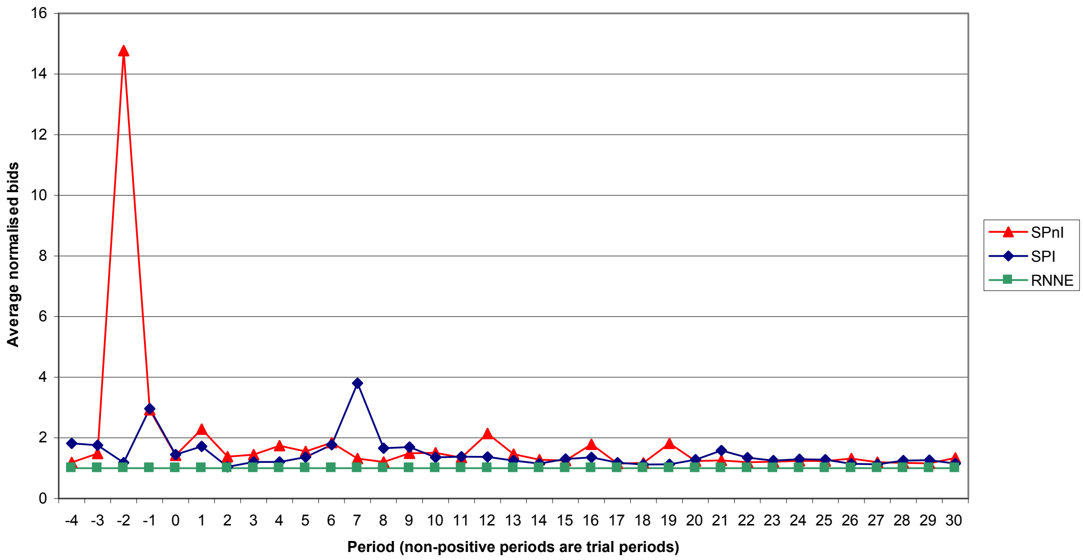

Figure 6.4. Average relative bids in SPI and SPnI.

6.3.2. Second-price sealed-bid auction

Figure 6.4 presents the average relative bids in the treatment SPI and $SPnI$ as well as the average bid predicted by Nash equilibrium of a standard model of a second price sealed bid auction ($RNNE$). The average bidders' behavior looks pretty the same in both treatments. The results of the Mann-Whitney U Test applied to subjects' average relative bids indicate that the two samples are not significantly different. In both of them, bidders used dominated strategy of biding above the valuation, which is consistent with the other existing studies (see Kagel, 1995).

Regret theory, presented in chapter 5 of this thesis, predicts that subjects bid their valuations in both treatments or overbid only in treatment SPI. This did not happen in the experiment. I wish to emphasize that the theoretical model assumes that subjects reached an equilibrium and this is not necessarily the case in the experiment. To understand the role of regret motive, I will now focus on the dynamic behavior.

In a second-price sealed-bid auction, a subject may experience two types of regret. First, when he loses at a price below his valuation, he may regret bidding too little. Second, when he wins at a price above his valuation, he may regretting bidding too high. In all the other situations, there is no clear regret. If a subject wins a good at a price below his valuation, there is nothing to regret. Similarly, a subject who loses at a price above his valuation has no reason to regret.

Figure 6.5 presents bidders reactions to the experienced conditions. The focus is on the following six conditions. First, a subject may win the auction at a price below his valuation, after bidding at least the valuation. Second, he may win the auction at a price below his valuation, after bidding below the valuation. Third, he can win at a price above his valuation. Fourth, he can lose at the price below his valuation. Fifth, he can lose at the price above the valuation, when bidding below the valuation. Sixth, he can lose at the price above the valuation, when bidding above the valuation. In SPI, a subject always knows which situation he faces. In SPnI, he does not distinguishes the last three situations. The reactions are counted for the following period. That is, if a subject finds himself in one of the six conditions and increases his relative bid in the following period, his reaction is counted as a "bid increase". If he experiences a given condition and he does not change his relative bid in the following period, his reaction is denoted as "no change". If he is in one of the six situations and decreases his relative bid, his reaction falls into category "bid decrease".

Most of the subjects changed their relative bids, suggesting that they did not reached equilibrium. Experienced regret affected decisions of the subjects. In both treatments, in above 70% experiences of winning at an unprofitable price leaded to a bid decrease. Above 85% of the experiences of losing at a profitable prices resulted in a bid increase. The reactions in SPI are stronger than in SPnI. When experiencing situations in which the role of regret is less clear, subjects' behavior was less consistent, but there seemed to be some convergence toward a dominant strategy of bidding the valuation. After bidding above the valuation and winning the good at the price below the valuation, most of the

Treatment	Experience condition	bid increase # (%)	no change # (%)	bid decrease # (%)	row total
SPI	Winning & $p<v_i\leq b_i$	69 (44%)	35 (22%)	54 (34%)	158 (27%)
	Winning $p<v_i$ & $b_i<v_i$	12 (70%)	0 (0%)	5 (30%)	17 (3%)
	Winning & $p\geq v_i$	15 (25%)	0 (0%)	46 (75%)	61 (11%)
	Losing & $p<v_i$	30 (91%)	0 (0%)	3 (9%)	33 (6%)
	Losing & $p\geq v_i>b_i$	27 (77%)	0 (0%)	8 (23%)	35 (6%)
	Losing, $p\geq v_i$ & $v_i\leq b_i$	85 (31%)	75 (27%)	114 (42%)	274 (47%)
	Total	238 (41%)	110 (19%)	230 (40%)	578
SPnI	Winning & $p<v_i\leq b_i$	79 (52%)	21 (14%)	51 (34%)	151 (29%)
	Winning $p<v_i$ & $b_i<v_i$	6 (55%)	0 (0%)	5 (45%)	11 (2%)
	Winning & $p\geq v_i$	18 (28%)	0 (0%)	47 (72%)	65 (13%)
	Losing & $p<v_i$	15 (88%)	0 (0%)	2 (12%)	17 (3%)
	Losing & $p\geq v_i>b_i$	17 (94%)	0 (0%)	1 (6%)	18 (3%)
	Losing, $p\geq v_i$ & $v_i\leq b_i$	101 (39%)	44 (17%)	113 (44%)	258 (50%)
	Total	236 (44%)	65 (13%)	219 (43%)	520
Grand total		474 (43%)	175 (16%)	449 (41%)	1098

Figure 6.5. Bidders' reactions to different experiences in SPI and SPnI.

subjects even further increased their relative bids, which does not seem rational at all. After bidding below the valuation and winning the good, most of the subjects increased their relative bids, which might be explained by discovering a dominant strategy. After bidding below the valuation and losing, most of the bidders increased their relative bids, which might be also attributed to discovering a dominant strategy. After bidding at least the valuation and losing, most of the subjects decreased their relative bids, which again looks as a sign of discovering a dominant strategy.

Figure 6.6 summarizes average reactions of the bidders. It is evident that regret affects bidders' behavior. A subject who had to pay above the valuation decreased his relative

	SPI	SPnI
Winning & $p<v_i\leq b_i$	48%	15%
Winning $p<v_i$ & $b_i<v_i$	26%	3%
Winning & $p\geq v_i$	-53%	-25%
Losing & $p<v_i$	27%	39%
Losing & $p\geq v_i>b_i$	30%	81%
Losing, $p\geq v_i$ & $v_i\leq b_i$	-29%	-18%

Figure 6.6. Average reactions of the bidders in SPI and SPnI

bid in the next round, so it seems that overpaying triggers regret which makes bidders less aggressive in the next auction. Losing at the price below the valuation resulted in more aggressive bidding in the next auction, so it looks like after bidding the valuation and losing, a subject regrets bidding too little and as a result bids more in the next auction. When experiencing situations in which the role of regret was less clear, subjects' behavior was less consistent and there seemed to be some convergence toward a dominant strategy of bidding the valuation. After bidding below the valuation, subjects rather decreased their bids. After bidding above the valuation, they rather increased their bids. The only unclear reaction was after bidding at least the valuation and winning the good for the price below the valuation. When this situation occurred, subjects on average increased their bids.

6.4. Conclusions

In standard auction theory, a bidder maximizes his monetary payoff to find an optimal bidding function. His behavior is unaffected by information feedback revealed after the auction. Regret theory extends the standard model by incorporating the anticipated regret in the utility function. In this model, the information feedback affects bidders' behavior. This paper studies regret motive in auctions in the laboratory setting. The experiment varies the information feedback in first- and second-price sealed bid auctions. For each type of the auction (first-price and second-price), there is a treatment in which the losing bidder learns the price and a treatment in which he does not. Changing the information feedback results in different bids in first price auctions but not in second price auctions. In a first-price auction, better informed bidders bid less aggressively. This behavior is not consistent with regret theory and is contrary to the results of Neugebauer and Selten (2006), who test the same question but in a different design. Although static behavior is not consistent with the regret motive, the dynamic behavior may be explained by the regret motive. After bidding below the valuation and seeing the price below the valuation, subjects increased their bids. After paying above the valuation, subjects decreased their bids. There was also some convergence to a dominant strategy in a second price auction.

6.5. Appendix - Instructions

Welcome to this experiment on decision-making. During the experiment, you and the other participants are asked to make decisions. If you make good decisions, you may earn money that will be paid off in cash together with a show-up fee of 2 EURO.

During the experiment, we ask you not to talk to each other. If you have a question, please raise your hand and an experimenter will assist you.

Description of periods

The experiment consists of 35 periods. The first five periods are trial periods. You will not earn money then. The following 30 periods are paid periods.

In each period, one good is auctioned off. The computer will be the auctioneer. There will be precisely three bidders: you and two automated bidders. You will only be interacting with these two automated bidders, not with the other human bidders in the room.

Each period consists of two phases:

- bidding phase
- outcome phase.

Bidding phase

Valuations

At the beginning of the bidding phase, you will be informed about your valuation. Your valuation represents the amount that we will pay to you if you win the auction. It will be each time randomly drawn from the interval [0,400] Eurocents. Each automated bidder will also have its valuation independently drawn from the interval [0,400] Eurocents.

Bids

After learning the valuation, you will be asked to submit a bid, which can be any integer between 0 and 1000, inclusive. The automated bidders will also independently and simultaneously submit their bids. Each automated bidder will bid so as to maximize its expected payoff, assuming that everyone else also maximizes the expected payoff.

Once all bids are collected, the bidding phase ends.

Outcome phase

Information

(In the treatments FPI and SPI, the following instruction on the information was provided:) In the outcome phase, you will get the following feedback on your screen: your status (winner or loser), your valuation, your bid, the price and your payoff.

(In the treatment FPnI and SPnI, the following instruction on the information was provided:) In the outcome phase, you will get the following feedback on your screen: your status (winner or loser), your valuation, your bid and your payoff. Additionally, if you win, you will learn the price.

Winners and losers

Your status (winner or loser) will depend on your bid. The bidder who submits the highest bid will win the auction. The other bidders will become losers. In other words, if your bid is higher than bids of the two automated bidders, you will win the good. Otherwise, you will not.

Payoffs

The payoff of the loser will be zero.

The winner will have to pay the price for the object. In other words, the payoff of the winner will be:

$$\text{payoff} = \text{valuation} - \text{price}.$$

(In the treatments FPI and FPnI, the following instruction was provided)

The price will equal the winner's bid. For example, if the following bids are submitted: 250, 110 and 301, the bidder whose bid is 301 will win the auction and will pay 301.

(In the treatments SPI and SPnI, the following instruction was provided)

The price will equal the second highest bid. For example, if the following bids are submitted: 250, 110 and 301, the bidder whose bid is 301 will win the auction and will pay 250.

This is the end of the instructions. If you have questions, please raise your hand.

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Samenvatting (Dutch summary)

Deze thesis is een verzameling van documenten op het gebied van veilingen. Deel I van deze thesis bestaat uit hoofdstukken 2, 3 en 4, die allen de veilingen op Internet bestuderen. Ik verklaar het fenomeen van 'sniping', een gemeenschappelijke praktijk van bieden op het laatste ogenblik. Andere auteurs stellen dat 'sniping' door onzekere bodtransmissie in het laatste deel van de veiling of onzekerheid in de waarde van het goed wordt veroorzaakt (zie Bajari en Hortacsu, 2004). Echter, met de huidige technologie van Internet, zien de bieders geen problemen met het verzenden van hun bod vlak vóór het eind van de veiling. Verder bieden vele Internetveilingen goederen met de bekende waarde aan de bieders aan.

Deel I richt zich ook op verdere vragen met betrekking tot de veilingen op Internet. Hoofdstuk 2 vergelijkt de verschillende regels ten aanzien van het beëindigen van veilingen op Internet. Hoofdstukken 2, 3 en 4 bestuderen het effect van het plannen van de veilingen op de definitieve toewijzing. Hoofdstuk 4 onderzoekt de relatie tussen mogelijke onvolmaaktheden van de transmissie van het bod en het biedgedrag.

Deel I stelt dat, om efficiency te ondersteunen, de openbare verkoper voor opeenvolgende of overlappende veilingen zou moeten opteren. Hoofdstuk 2 toont verder aan dat het hebben van een vaste tijd om de veiling te beëindigen in plaats van een flexibele eindtijd goed voor de efficiëncy is. Tot slot tonen alle hoofdstukken aan dat het fenomeen van 'sniping' aan de multipliciteit van veilingen met het zelfde dienstenaanbod kan worden toegeschreven.

Deel II van deze thesis onderzoekt de rol van het terugkoppelen van informatie bij het bieden op het van gedrag van teleurgestelde bieders in verzegelde-bodveilingen. Hoofdstuk 5 behandelt een theoretische studie van het effect van informatieterugkoppeling op te voorziene spijt en vreugde op het biedgedrag in verzegelden-bodveilingen. Ik toon aan dat een speler die bereid is tot het inleveren van financiële garantie om spijt te vermijden en vreugde te maximaliseren agressiever zal bieden dan de standaardtheorie voorstelt.

Het gedrag hangt van de terugkoppeling van informatie af, aangezien een bieder spijt of vreugde slechts dàn ervaart als hij zich van de gemiste kansen bewust is.

Hoofdstuk 6 rapporteert over de resultaten van een experiment omtrent de relatie tussen openbare bekendmaking op de prijs en het gedrag van bieders in de veilingen van het privé-waarde ver*zegelde-bod. Het waargenomen dynamische gedrag wordt beïnvloed door de ervaring van spijt, welke beurtelings met de teruggekoppelde informatie verwant is.